A MODEL OF ECONOMIC GROWTH [1]

THE purpose of a theory of economic growth is to show the nature of the non-economic variables which ultimately determine the rate at which the general level of production of an economy is growing, and thereby contribute to an understanding of the question of why some societies grow so much faster than others. There is general agreement that the critical factors determining the trend rate of growth are to be sought in the savings propensities of the community (which determine the rate of capital accumulation), the flow of invention or innovation (which determines the rate of growth of productivity) and the growth of population. Until recently, these factors were regarded as the parameters of a growth model—*i.e.*, as non-economic variables which are invariant with respect to changes in the other variables—and theoretical inquiry was confined to the more modest task of showing the particular relationships that must prevail between the values of these different parameters in order that they should be consistent with a steady rate of growth for the economy as a whole. But more recently, there has been an increasing awareness of the fact that neither the proportion of income saved nor the rate of growth of productivity per man (nor, of course, the rate of increase in population) are independent variables with respect to the rate of increase in production; and that the actual rate of progress of a capitalist economy is the outcome of the mutual interaction of forces which can adequately be represented only in the form of simple functional relationships (like supply or demand curves) rather than by constants. The purpose of this paper is to present a simple model of economic growth based on a minimum number of such relationships.[2]

A satisfactory model concerning the nature of the growth process in a capitalist economy must also account for the remarkable historical constancies revealed by recent empirical investigations. It was known for some time that the share of wages and the share of profits in the national income has shown a remarkable constancy in " developed " capitalist economies of

[1] This paper owes a great deal to discussions with Professor D. G. Champernowne, both in its general ideas and even more in the detailed presentation of the mathematical parts of the argument. I am particularly indebted for his help in working out the implications of the assumptions in mathematical terms, and for the mathematical proofs of some of the propositions made, though he bears no responsibility for the choice of assumptions underlying the models.

[2] The present paper represents an elaboration and further development of ideas put forward in two earlier papers by the author, " Alternative Theories of Distribution," *Review of Economic Studies*, Vol. XXIII, No. 2 (March 1956), pp. 94–100, and " Capitalist Evolution in the Light of Keynesian Economics," a lecture delivered at the University of Peking in May 1956, and reprinted in *Sankhyā* (*Journal of the Indian Statistical Institute*), Vol. 18, Parts 1 and 2, June 1957. Reference may perhaps also be made to two former papers by the author which set out the reasoning underlying some of the arguments and assumptions in more detail, " Mr. Hicks and the Trade Cycle," ECONOMIC JOURNAL, December 1951, pp. 833–844, and the "The Relation of Economic Growth and Cyclical Fluctuations," ECONOMIC JOURNAL, March 1954, pp. 53–71.

the United States and the United Kingdom since the second half of the nineteenth century.[1] More recent investigations have also revealed that whilst in the course of economic progress the value of the capital equipment per worker (measured at constant prices) and the value of the annual output per worker (also in constant prices) are steadily rising, the trend rates of increase of both of these factors has tended to be the same, so as to leave the capital/output ratio virtually unchanged over longer periods.[2] This means that while progress involved a continuous increase in the amount of capital used per worker—whether capital is measured in terms of an index number of the value of capital goods at constant prices, in terms of horse-power per man or in tons of steel embodied in equipment per operative, etc.—there was no "deepening" of the capital structure in the economist's sense: no increase in the amount of "waiting" per unit of current output, or in the ratio of "embodied" to "current" labour, or in the length of some (arbitrarily measured) "investment period." [3] Constancy in the share of profit and in the capital/output ratio also involves constancy in the rate of profit earned on investments (in the "marginal efficiency" of capital), and this again appears to be confirmed by empirical investigations.[4] Existing theories are unable to account for such constancies except in terms of particular hypotheses (unsupported by any independent evidence), such as the unity-elasticity of substitution between Capital and Labour,[5] or more recently, con-

[1] For the United Kingdom the share of wages has shown only small variations around the level of 40% of the national income in the period 1840–1950. (Cf. Phelps Brown and Hart, "The Share of Wages in the National Income," ECONOMIC JOURNAL, June 1952, pp. 253–77.) In the United States the share of wages and salaries remained constant at around 60% of the national income up to about 1929, and has since shown a rising trend, mounting to 69% in the post-war decade, while the share of profits and property income (dividends, interest and rent) declined since 1929 from 38 to around 30%. Some of this change no doubt reflects the rise in wage and salary earners as a proportion of the total population. (Cf. Kuznets, "Long Term Changes in the National Income of the U.S.A. since 1870," *Income and Wealth*, Series II.)

[2] According to Mr. Maiwald's calculations based on fire-insurance figures, the capital/output ratio in Britain remained practically unchanged in the period 1870–1914 (at around 3·3) and fell slightly (to around 3·0) in the period 1914–38 (*Economic History Review*, Part I, 1956, p. 102). The same impression is gained from the study of Phelps Brown and Weber ("Accumulation, Productivity and Distribution in the British Economy, 1870–1938," ECONOMIC JOURNAL, June 1953, pp. 263–88) for the period 1870–1900, though they indicate a rising ratio for the period 1900–14, and a falling ratio for 1924–38. In the United States the capital/output ratio has shown a slightly rising trend from the decade 1879–88 to the decade 1909–18, and a falling trend since, and (ignoring the depression period) is not significantly different now (at around 3·0) than it was sixty years ago. (Cf. Fellner, *Monetary Policies and Full Employment*, Table 3, p. 80), based on Kuznets' estimates.)

[3] The usual explanation for this apparent paradox is, of course, that the productivity of labour increased at the same rate in the capital-goods-making industries (taken as a group) as in the economy as a whole (*i.e.*, in all industries taken together) as a result of the peculiar character of technical progress (the "neutrality" of inventions). As will be shown below however no such assumption is necessary for explaining the constancy of the capital/output ratio.

[4] According to Phelps Brown and Weber (*loc. cit.*) the rate of profit on capital in the United Kingdom (including buildings) remained remarkably steady at around $10\frac{1}{2}\%$ in the period 1870–1914, the annual variations being within the range of $9\frac{1}{2}$–$11\frac{1}{2}\%$. The same steadiness is shown in the United States in the relationship of property income to total capital (cf. Kuznets, *loc. cit.*)

[5] Hicks, *Theory of Wages* (1932) Ch. VI, *passim*.

stancy of the degree of monopoly or the " neutrality " of technical progress.[1] One of the merits of the present model is that it shows that the constancy in the capital/output ratio, in the share of profit and in the rate of profit can be shown to be the consequence of endogeneous forces operating in the system, and not just the result of some coincidence—as, *e.g.*, that " capital saving " and " labour saving " inventions happened (historically) to have precisely offset one another, or that the growth in monopoly happened (historically) to have been counterbalanced by the fall in raw-material prices in terms of finished goods.[2]

Basic Properties of the Model

The present model is based on Keynesian techniques of analysis and follows the well-known " dynamic " approach originally developed by Mr. Harrod [3] in regarding the rates of change of income and of capital as the dependent variables of the system. The properties of our model differ, however, in important respects from those of Mr. Harrod and other writers, and these differences can be traced to the following:

(1) It is assumed here that in a growing economy the general level of output at any one time is limited by available resources, and not by effective demand. The model in other words assumes " full employment " in the strictly Keynesian sense—a state of affairs in which the short-period supply of goods and services in the aggregate is inelastic and irresponsive to further increases in monetary demand. This need not necessarily imply the full employment of labour except in a developed economy where the available capital equipment is sufficient or more than sufficient to employ the whole of the available working force. But it does imply that, excepting for periods in which the process of growth through capital accumulation (for reasons outlined later) is altogether interrupted, the system cannot long operate in a state of (Keynesian) under-employment equilibrium, because at any level of output short of " full employment " the aggregate demand associated with that particular level of output will exceed the aggregate supply price of that output, and thus lead to an expansion in output until a state of full employment is reached. In a state of full employment, on the other hand, aggregate demand and aggregate supply (in real terms) are brought into equality through the movement of prices in relation to prime costs, *i.e.*, the relation of prices to wages. (It is assumed that any rise in prices in relation to wages increases savings relative to investment and thus reduces aggregate

[1] Kalecki, *Essays in the Theory of Economic Fluctuations*, Chapter I; Harrod, *Towards a Dynamic Economics*, p. 23; Joan Robinson, *The Rate of Interest and Other Essays*, pp. 90–97, and *The Accumulation Capital*, pp. 73–100.

[2] Kalecki, *op. cit.*, pp. 32–34.

[3] " An Essay in Dynamic Theory," Economic Journal, March 1939, subsequently elaborated in *Towards a Dynamic Economics* (Macmillan, 1949). Substantially the same analysis was put forward in the well known article by Domar (*Econometrica*, 1946).

demand in real terms, and vice versa).[1] The assumption that there can be no under-employment equilibrium in periods in which the rate of growth of capital and income is normal is not arbitrary; it is based on the view that an equilibrium of steady growth is inconsistent with under-employment equilibrium. For in the latter situation the relationship of prices and wages is determined by extraneous factors (such as the " degree of monopoly," full-cost pricing, Marshall's " traditional margin of profit on turnover " or what not) and, as will be shown below, these either yield an insufficient ratio of savings to income (in which case, in a growing economy, the system is not stable *below* full employment) or else an excessive ratio of savings to income (in which case, for reasons described by Mr. Harrod,[2] the process of growth cannot continue, and the system will relapse into stagnation). A state of Keynesian under-employment equilibrium, whilst it is perfectly consistent with a static short-period equilibrium, is therefore inconsistent (except by a fluke) with a dynamic equilibrium of steady growth.[3]

The Keynesian techniques of analysis were originally designed, of course, to explain how an economy can remain indefinitely in a state of under-employment equilibrium; it may seem at first sight paradoxical therefore to label a model " Keynesian " if it is based on the full-employment assumption. However, as I argued in an earlier paper [4] the specifically Keynesian apparatus of thought can be applied to full-employment situations and not only to under-employment situations, and there is some evidence that in an earlier stage in the development of his ideas (in *A Treatise on Money*) Keynes applied the multiplier principle—the idea that is, that expenditure decisions determine income and savings rather than the other way round—for the purpose of a price theory rather than an employment theory, even though in the *General Theory* he explicitly disclaimed that his ideas had any relevance to full-employment conditions.[5] Yet the specifically Keynesian hypothesis that

[1] Any sharp and clear-cut distinction between an under-employment equilibrium, where production is limited by effective demand, and a full-employment-equilibrium, where it is limited by available resources, presumes, of course, something like a reverse L-shaped supply function for output as a whole, so that marginal costs are either infinite (or indefinite) or else equal to average prime costs, and thus in neither case exert an important influence on the ruling relationship between prices and wages. In reality there is a twilight zone of semi-full employment in which the supply of goods and services in the aggregate is neither elastic nor inelastic, and in which an increase in money demand will increase both prices and production in roughly equal measure. For the purposes of the present model it will be assumed that this " twilight zone " is sufficiently narrow in range to be left out of consideration altogether. (Its introduction would complicate the exposition without radically altering the basic features of the model.)

[2] " An Essay on Dynamic Theory," *loc. cit.*, pp. 23–26; *Towards a Dynamic Economics*, pp. 77 ff.

[3] I believe it was Mr. Harrod's failure to see the inconsistency between a continuous under-employment equilibrium and a state of steady growth which led him to the belief that a dynamic equilibrium of growth is necessarily unstable. His proposition applies only if the savings coefficient is extraneously determined, and this in turn presumes a state of affairs where short-period output is elastic. It does not apply to full employment or even to the " twilight zone " of semi-full employment.

[4] " Alternative Theories of Distribution," *loc. cit.*, p. 94.

[5] *General Theory*, pp. 3, 26 and 112.

equilibrium between savings and investment is secured through a movement of prices and/or incomes, rather than through changes in the rate of interest, is just as fruitful in the context of a dynamic growth model based on the postulate of full employment as in a (short-period) static model based on the postulate of under-employment; nor does the postulate of full employment appear as unrealistic at the present day (at any rate in the context of a dynamic theory of growth) as it appeared in the 1930s. Although the great depression of the 1930s was both more severe and more prolonged in duration than its predecessors, the gloomy forebodings made at the time that it heralded the approach of an era of long-term economic stagnation have certainly proved premature: since 1945 the momentum of growth in the capitalist economies has been at least as strong as in any comparable period since 1870. It does not therefore seem unrealistic to assume that capitalist economies operate under full-employment conditions in all such periods (and these appear to take up the majority in terms of chronological time) when capital is accumulating and the national income is growing. Similarly, in the case of under-developed economies, whilst there may be vast numbers of unemployed or under-employed the situation is not one of Keynesian under-employment as the supply of goods is in general inelastic in the short period and irresponsive to increases in monetary demand. Here again it certainly appears more correct to assume that output at any one time is limited by the scarcity of resources rather than by effective demand.

(2) The second main respect in which the present model departs from its predecessors is that it eschews any distinction between changes in techniques (and in productivity) which are induced by changes in the supply of capital relative to labour and those induced by technical invention or innovation—*i.e.*, the introduction of new knowledge. The use of more capital per worker (whether measured in terms of the value of capital at constant prices, in terms of tons of weight of the equipment, mechanical power, etc.) inevitably entails the introduction of superior techniques which require " inventiveness " of some kind, though these need not necessarily represent the application of basically new principles or ideas. On the other hand, most, though not all, technical innovations which are capable of raising the productivity of labour require the use of more capital per man—more elaborate equipment and/or more mechanical power. Hence the speed with which a society can " absorb " capital (*i.e.*, it can increase its stock of man-made equipment, relatively to labour) depends on its technical dynamism, its ability to invent and introduce new techniques of production. A society where technical change and adaptation proceed slowly, where producers are reluctant to abandon traditional methods and to adopt new techniques is necessarily one where the rate of capital accumulation is small. The converse of this proposition is also true: the rate at which a society can absorb and exploit new techniques is limited by its ability to accumulate capital.

It follows that any sharp or clear-cut distinction between the movement *along* a " production function " with a given state of knowledge, and a *shift* in the " production function " caused by a change in the state of knowledge is arbitrary and artificial.[1] Hence instead of assuming that some given rate of increase in productivity is attributable to technical progress which is superimposed, so to speak, on the growth of productivity attributable to capital accumulation, we shall postulate a single relationship between the growth of capital and the growth of productivity which incorporates the influence of both factors. The plausible shape of this " technical progress function " is that given by the curve TT' in Fig. 1. Let C_t and O_t represent the capital per worker and the annual output per worker at time t, so that $\frac{1}{C_t}\frac{dC}{dt}$ (measured horizontally) represents the annual percentage growth in capital per worker, and $\frac{1}{O_t}\frac{dO}{dt}$ (measured vertically) the annual percentage growth in output per man. The shape and the position of the curve reflect both the magnitude and the character of technical progress as well as the increasing organisational, etc., difficulties imposed by faster rates of technical change. It may be assumed that *some* increases in productivity would take place even if capital per man remained constant over time, since there are always some innovations—improvements in factory lay-out and organisation, for example—which enable production to be increased without additional investment. But beyond these the growth in productivity will depend on the rate of growth in the capital stock—clearly the more capital is increased, the more labour-saving technical improvements can be adopted, though there is likely to be some maximum beyond which the rate of growth in productivity could not be raised, however fast capital is being accumulated. Hence the TT' curve is likely to be convex upwards and flatten out altogether beyond a certain point. The postulate of the existence of a given curve presumes, of course, a constant flow in the rate of new ideas over time. Variations in the flow of new ideas, and in the readiness with which they are adopted, are likely to be reflected in shifting the height of the curve rather than in altering its general character.[2] In an unprogressive economy, with a low capacity to absorb technical change the height of the TT' curve will be relatively low (as in the dotted line in Fig. 1), whilst important new discoveries (such as the invention of the internal combustion engine or atomic energy) are likely to raise the position of the curve considerably for some time.

P represents the particular point on the TT' curve where it is crossed by

[1] The technical possibilities shown by the " production function " of any one period are in fact nothing more than the reflection of the yet unexploited inventions and innovations of the past.

[2] Our TT' curve thus reflects not only " inventiveness " in the strict sense, but the degree of technical dynamism of the economy in a broader sense—which includes not only the capacity to think of new ideas, but the readiness of those in charge of production to adopt new methods of production.

a line drawn from the origin at an angle of 45 degrees—where, in other words, the percentage rate of growth of capital and the percentage rate of growth of output are equal. When the rate of capital accumulation is less than this rate the percentage rate of growth in output will exceed the growth in capital (involving a fall in the capital/output ratio), and vice versa.

The recognition of the existence of a functional relationship between the proportionate growth in capital and the annual proportionate growth in productivity shows the futility of regarding the movements in the capital/

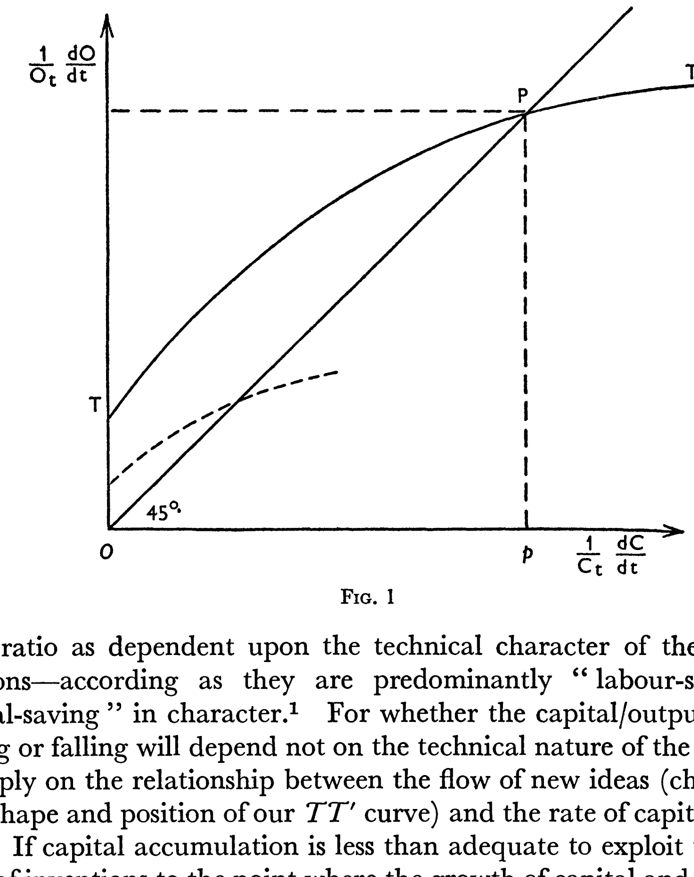

Fig. 1

output ratio as dependent upon the technical character of the stream of inventions—according as they are predominantly " labour-saving " or " capital-saving " in character.[1] For whether the capital/output ratio will be rising or falling will depend not on the technical nature of the inventions, but simply on the relationship between the flow of new ideas (characterised by the shape and position of our TT' curve) and the rate of capital accumulation. If capital accumulation is less than adequate to exploit the current stream of inventions to the point where the growth of capital and the growth in output are equal—if the actual position is to the left of P—the capital/output ratio will be falling, and the character of inventions will appear to be predominantly " capital-saving " in nature; if the position is to the right of P they will appear to be predominantly " labour-saving " in nature.[2] As will be shown below, with a given TT' curve, the system will always tend

[1] Cf. Harrod, *op. cit.*, p. 23; Joan Robinson, *op. cit.*, pp. 164 ff.

[2] New techniques capable of raising the productivity of labour in any given proportion will, of course, be all the more profitable the less additional capital they require for adoption. It is evident, therefore, that relatively labour-saving and capital-using innovations are the more likely to be adopted the higher the rate of capital accumulation.

towards the point where the growth of capital and the growth in productivity are equal—P is therefore the long-run equilibrium point and Op the long-run equilibrium rate of growth. (The basic reason for this is that if the rate of capital accumulation is less than Op, it will tend to be stepped up and the rate of profit on new investment will rise over time; in the converse case, the rate of growth of capital will tend to be slowed down, and the rate of profit will fall. It is only when capital and output grow at the same rate, and the capital/output ratio is constant, that the rate of profit remains constant over time; and then technical progress will appear to take on a "neutral" character.[1]) An upward shift in the TT' curve caused by a burst of new inventions will cause (for a time) a falling capital/output ratio; progress will thus appear to take on a predominantly "capital-saving" character. A drying up in the flow of ideas, represented by a downward shift in the TT' curve will, on the other hand, cause the growth of productivity to lag behind; the capital/output ratio will rise, and innovation will appear to be predominantly "labour-saving."

This is not to deny that *in particular industries* the capital/output ratio may change significantly in an upward or downward direction as a result of the character of new inventions occurring there. If an important capital-saving invention occurs in some industry—due, for example, to some revolutionary improvement in design which raises the productive capacity of capital equipment of a given value—the capital/output ratio in that industry is bound to fall. But precisely because this involves a gradual rise in the rate of profit, and thus in the rate of capital investment, it will induce compensating changes in the capital/output ratio of other industries, so that for the economy as a whole (and in the long run) the overall capital/output ratio will tend to remain constant.

(3) Our model, like other macro-economic models, is based on simple aggregative concepts of income, capital, profits, wages, investment and savings, expressed in real terms—*i.e.*, in terms of values of constant purchasing power. This raises all the familiar difficulties involved in the use of index numbers, as well as the more fundamental questions of the measurement of the quantity of capital. The measurement of the stock of capital in terms of money values corrected for price changes raises peculiar problems of its own on account of: (i) the fact that owing to technical progress, the capital goods produced in any one period are physically non-identical with the capital goods produced in previous periods, and therefore cannot simply be *added* to the latter, even if the proportions of the different capital goods serving different end-uses remained the same; [2] (ii) that the *value* of the stock

[1] cf. pp. 609–10 below.

[2] Much the same problem arises, of course, in the measurement of real income, since in a progressive economy new kinds of commodities are constantly being introduced. But whereas in the case of income, the changing character and composition of the constituents could be ignored in the first approximation (by assuming, *e.g.*, a single type of consumption good, such as "bread") in the case of capital goods it cannot, since it is the essence of a change of techniques that the character of the instruments, etc., used in production changes.

of capital existing at any moment is not the sum of the values of the capital goods produced in the past, but is that quantity *less* accrued depreciation.

Whatever may be the situation as regards the "stationary states" of neo-classical theory, there can be no question that in a developing economy with constantly changing knowledge and techniques neither the problem of equating the capital goods produced at different dates and at differing states of knowledge nor the measurement of depreciation admits of any clear-cut theoretical solution: the measurement of the stock of *real* capital must therefore necessarily be based on some (more or less arbitrary) convention. One such convention would measure the stock of real capital in terms of the amount of mechanical power which the outstanding stock of capital goods (or the addition to it in a particular year) represents. Another such convention (which appears rather less arbitrary) measures it in terms of the total weight of steel embodied in the capital equipment.[1] For the purposes of this model we shall adopt the latter convention. We shall also assume that the average price, per ton of steel, of finished capital goods produced in successive periods remains constant in terms of income units so that the rate of growth of capital is the same in "steel" as in terms of real income. (This assumption, while it greatly simplifies the exposition, is not an essential feature of the model.) With regard to depreciation, we shall assume: (*a*) that individual capital goods retain their physical efficiency until they are scrapped; (*b*) that the proportion of the outstanding capital stock which is scrapped in any one year (again measured in tons of steel) is a constant fraction of the total stock of capital. Hence we shall define depreciation as the value of that part of the output of capital goods produced in any one period which is needed to maintain the total weight of steel in the outstanding capital stock constant (and which, in a steadily growing economy, is also a constant fraction of total turnover). We shall define income, savings, investment *net* of depreciation in this special sense of the term "net." The difference between gross and net income and gross and net savings will be assumed to be identically equal to the difference between gross and net investment.[2]

(4) It is evident from the foregoing that the prime mover in the process of economic growth is the readiness to absorb technical change combined with the willingness to invest capital in business ventures. It is through the continued increase in the amount of machinery, etc., used in combination

[1] Another variant of this kind of measurement would be the total physical weight of all capital goods produced (of whatever material it consisted).

[2] It would also be possible, of course, to value capital at the original cost of installation of capital goods less accrued depreciation (the latter being spread over the physical life of the assets, either by the straight-line method, the reducing-balance method, etc.) and deflated by an index of capital-goods prices. But the point is that the application of a price index to the historical cost of the capital goods produced at different periods implicitly makes use of some kind of convention of the type introduced here explicitly. (Measuring capital in terms of value deflated by capital-goods-prices is not the same, of course, as valuing capital in real income units; but the complications due to the differences between the two will not be gone into here.)

with labour that the productivity of labour is continuously increased. In a capitalist economy the process of accumulation is the resultant of innumerable investment decisions made by entrepreneurs—using this term in a broad sense, so as to comprise the owners of risk capital generally, as well as the Boards of Directors of companies who take the actual decisions on their behalf. Any act of investment the outcome of which is necessarily uncertain at the time the decisions are taken, implies an act of faith—it involves a favourable judgment concerning the future course of markets, as well as the future relationship of prices and costs. Unless entrepreneurs are willing to revise their estimates of future sales and profits upwards in the light of current experience—unless they are imbued with sufficient optimism to react favourably to favourable events, and to increase the amount of capital invested in response to an increase in current sales and profits—it is difficult to envisage the growth of production and capital as a continuous process. For unless the capacity to produce is continually increased, the increase of production must necessarily come to a halt; a continued growth in output capacity presupposes in turn a belief (which must be grounded, and can only be grounded, in past experience) in the continued growth in markets. In order that there should be continued growth, it is necessary therefore to suppose both that, on the one hand, output increases as a result of capital investment and, on the other hand, investment takes place in response to an increase in output. Hence as a complement to our technical progress function which denotes the former we must postulate a function based on assumptions concerning entrepreneurial psychology to denote the latter. With regard to this second function, which we shall call the investment function, it will be assumed: (i) that given the (expected) rate of profit on capital, entrepreneurs desire to maintain a constant relationship between the amount of capital invested and their turnover; (ii) that this relationship between desired capital and turnover is an increasing function of the expected rate of profit on capital; [1] (iii) that the investment decisions of each "period" are governed by the condition that actual capital is to be brought into line with desired capital, the length of the "period" being so defined as to make it technically feasible to eliminate in one period the backlog of investment (the difference between desired and actual capital) existing at the beginning of the period; [2] (iv) that entrepreneurs expect the same growth in turnover

[1] The assumption that for each enterprise there is some *desired* amount of invested capital in relation to turnover which is itself a rising function of the rate of profit can be justified by the greater risk and uncertainty of expectations for the more distant future as against the nearer future and the consequent preference (at equal rates of expected return) for investments with a more rapid turnover of capital as against investments which entail a longer period of inevitable commitment. A high capital/output ratio implies a longer period of commitment because it implies a higher ratio of fixed capital to circulating capital (irrespective of any differences in the durability of fixed capital).

[2] As we shall see later (cf. pp. 619–20 below) this assumption is applicable only to an "advanced" capitalist economy in which, at the ruling rate of profit, the capital stock in the various industries has already been brought into the desired relationship with turnover, so that net investment takes place only in response to a rise in the (expected) turnover or a rise in the rate of profit.

in the coming period as was actually attained in the previous period; (v) that they expect to obtain the same margin of profit on turnover in the coming period as actually obtained in the previous period.

These assumptions imply an investment function which makes investment of any period partly a function of the change in output in the previous period [1] and partly of the change in the rate of profit on capital in that period. They are not, of course, the only possible assumptions that one could choose in this regard. It would be possible to assume, for example, that investment, at any given rate of profit, is a constant percentage addition to the existing stock of capital, rather than a coefficient related to the increase in turnover.[2] But in a model which makes the amount of profits actually generated in the production process dependent on the rate of investment and makes the rate of investment in turn dependent on the growth of profits, it is necessary to postulate a certain minimum " buoyancy " in entrepreneurial behaviour in order to ensure that the investment necessary to generate the profits which call forth a further increase in investment in the next period actually *does* take place, so that productivity, total output, profits and investment continue to grow. Without assuming a certain minimum of " buoyancy," the mere accrual of fresh investment opportunities through technical progress will not alone ensure the continued growth in production [3]—since the latter requires in addition that effective demand and profits should

[1] In earlier articles on the trade cycle (cf. ECONOMIC JOURNAL, March 1940, p. 79, and ECONOMIC JOURNAL, December, 1951, p. 837) I strongly criticised the use of the acceleration principle in connection with trade-cycle models, because it assumes a constant relationship between output and capital (or rather between output and output capacity), whereas the recognition of a changing relationship between these two seemed to me essential for an understanding of the cyclical mechanism. I recognised, however, the validity of the principle " as between alternative positions of long period equilibria " (*ibid.*, p. 838). For the purpose of a long-run model of economic growth it is legitimate to divide time into " periods " long enough for the capital stock in any one period to be fully adjusted to the output expected for that period at the beginning of that period; which means that the " acceleration principle " is an appropriate principle to apply to characterise investment behaviour for such " periods."

[2] On the implication of this latter assumption cf. p. 610, note 1 below. The difference between these two kinds of assumptions relating to investment behaviour may be explained as follows. The postulate of an investment function where the rate of increase of the stock of capital (*i.e.*, the rate of investment as a proportion of the existing stock of capital) is treated as an increasing function of the rate of profit on capital implies the assumption that entrepreneurial risk is an increasing function of the rate of capital accumulation, irrespective of the relation between the growth of the capital stock and the growth of turnover. It thus corresponds to Keynes' declining marginal-efficiency-of-capital function, as re-interpreted by Kalecki's principle of increasing marginal risk. The postulate of an investment function where the desired stock of capital in relation to turnover is treated as an increasing function of the rate of profit on capital implies, on the other hand, that the principle of increasing risk is not applicable to the rate of capital accumulation as such, but only to a situation where the (proportionate) rate of capital accumulation exceeds the rate of growth of turnover. This seems a more reasonable supposition for so long as the growth of capital merely keeps pace with the growth of turnover there seems to be little empirical justification for the belief that a faster rate of growth of capital entails a higher subjective marginal risk to the entrepreneur. This latter assumption, moreover, as will be shown below, is consistent with a stable equilibrium of steady growth whereas the former is not.

[3] Cf. ECONOMIC JOURNAL, December 1951 p. 842.

increase sufficiently to match the growth in potential supply, and thus keep the process of accumulation going.

(5) We shall assume that monetary policy plays a purely passive role—which means that interest rates, subject to differences due to borrowers' risks, etc., follow, in the long run, the standard set by the rate of profit obtainable on investments. The operation of our model is consistent with continued price-inflation (with money wages rising faster than productivity) or with a constant price level (money wages rising *pari passu* with productivity). It is in principle also consistent with constant money wages (money prices falling with the rise in productivity), though the latter situation might give rise to additional complications as regards investment behaviour which will not be gone into here. We shall deal with the case of isolated communities only—*i.e.*, we shall ignore the problems arising in connection with trade between regions in differing stages of development.

(6) We shall ignore also the influence of a change in the share of profits and wages and of a change in rate of profit on capital (or of interest rates) on the choice of techniques adopted which has been the focal point of attention of neo-classical theory.[1] At any one time the individual entrepreneur has, of course, a variety of " techniques " to choose from, and he may be assumed to choose the particular technique which secures the lowest cost, or the highest rate of return, on his investment. It seems reasonable to assume, however, that the choice of technique is far more dependent on the prevailing prices of different types of capital goods and on the price of labour in terms of commodities generally (in so far as this reflects the productivity of labour) than on the prevailing rate of profit or the prevailing interest rates. If an entrepreneur in an advanced economy employs bulldozers for making roads, whilst his opposite number in an under-developed country employs only shovels, this is not, to any significant degree, the consequence of differences in the prevailing rates of profit (or of the rates of interest on loans) in the two communities, but simply of the fact that the price of bulldozers in terms of shovels is much lower in the advanced community than in the primitive community. As an economy progresses as the combined result of capital accumulation and technical progress the prices of " superior " capital goods (capable of securing a higher output per unit of work) may be assumed to fall continuously relative to inferior capital goods: the character

[1] The importance attributed in neo-classical theory to the choice between more or less labour-saving and capital-using techniques being dependent on the rate of profit (or the rate of interest) is, of course, inherently connected with the acceptance of the marginal productivity theory as the basic principle in the explanation of the pricing process and the determination of distributive shares: for it is the choice among a range of techniques requiring more or less capital (and less or more labour) per unit of output which alone makes it possible (theoretically) to assign specific marginal productivities to the factors of production, Capital and Labour, in the long run. Our model, however, shows that for the system as a whole the share of wages and of profits, and the rate of profit on capital is determined quite independently of the principle of marginal productivity; the choice of alternative methods of production differing in " capital intensity " at any *given* state of knowledge loses therefore its central importance in the theoretical scheme.

of the most economical type of equipment will continually alter. This alone is sufficient to explain why it becomes profitable to employ bulldozers once a certain level of capital accumulation has been attained which has not been profitable at a lower stage of accumulation.[1] If, in addition, the use of the bulldozer-technique involves a higher investment per unit of output (a higher capital/output ratio) than the shovel-technique (which is by no means necessarily the case!) it will also be true that the rise in wages in terms of commodities, associated with the growth in the productivity of labour, will contribute to making the bulldozer-technique profitable at a certain stage of development.[2] In the latter case it may further be true that the use of bulldozers will be stimulated not only by the rise in wages due to the growth of productivity, but also by a rise in the *share* of wages at any given level of productivity due to a fall in profits. Thus as between two growing communities with equal technical progress functions, but in one of which the rate of profit is lower than in the other (due to a higher propensity to save out of profits and/or wages) the introduction of bulldozers may occur at a somewhat earlier stage of development (at a lower level of productivity and capital accumulation) than in the high-profit community, which in turn may have repercussions on the speed of development—at any rate during an intermediate stage, until an equilibrium of steady growth is attained. But we shall regard these as secondary complications which can safely be neglected in the first approximation; which means that we shall regard the choice of techniques as entirely a matter of the relative prices of different types of capital goods, which can be assumed to alter with the accumulation of capital and the progress of techniques in the capital-goods making industries.

[1] It may be thought that, given international trade, a particular under-developed country, in accordance with the law of comparative costs, would export shovels and import bulldozers and thereby gain the same terms of choice (as between different techniques) as the advanced country. But this pre-supposes a potential market for inferior capital goods in the advanced country (as well as a potential market for superior capital goods in the primitive country) which is non-existent.

[2] The fall in the price of bulldozers in terms of shovels will, of course, reduce the capital/output ratio involved in the bulldozer-technique relatively to the shovel-technique; and it may be supposed that at a certain stage of development the capital/output ratios in the two techniques will become identical. But whether this stage will have been reached or not at the time when the introduction of the bulldozer technique becomes profitable will depend (in part) on the rate of growth of productivity in the bulldozer-making industry relative to the rate of growth in the productivity of labour in the economy in general. The lower the former is relative to the latter, the more its introduction will have been prompted by the rise in wages (in terms of commodities in general) and the less by the fall in the price of bulldozers relative to shovels. If the introduction of the bulldozer technique has in the main been due to the rise in wages, and not to the fall in its price in relation to shovels, its introduction is in the nature of a " compensating " labour-saving innovation prompted by the introduction of capital-saving innovations in other parts of the economy. The fact that in the course of progress the prices of " superior " capital goods fall continually in relation to relatively inferior capital goods is not, of course, inconsistent with our assumption (stated on p. 599 above) that the price of newly produced capital goods per ton of steel embodied remain constant in terms of consumption goods.

The Working of the Model

We are now in a position to set out the essential features of the model. We shall examine its mode of operation in two stages—first, under the hypothesis of a constant working population and second by allowing for population growth. In the former case (constant full employment being assumed) the proportionate rate of growth in total real income, Y_t, will be the same as the proportionate rate of growth in output per head, O_t. In the latter case the proportionate change in total real income will be the sum of the proportionate change in productivity, O_t, and the proportionate change in the working population, L_t.

(a) *Constant Working Population*

We shall postulate: (i) given savings propensities for profit-earners and wage-earners, respectively;[1] (ii) that the investment decisions in any one period are governed by the desire to maintain the capital stock in a given relationship to turnover, modified by any change in the rate of profit on capital; (iii) a given technical relationship between the (proportionate) rate of growth in productivity per man and the (proportionate) rate of growth in capital per man. Writing Y_t, K_t, P_t, S_t, I_t for real income, capital, profits, savings and investment at the time t, we may assume the familiar income identities

$$S_t \equiv I_t \equiv K_{t+1} - K_t$$

To represent our three relationships mentioned above we shall, for expository purposes, adopt linear equations as follows:

(1) *Savings Function*

$$S_t = \alpha P_t + \beta(Y_t - P_t)$$

where $$1 > \alpha > \beta \geqslant 0$$

(2) *Investment Function*

(2.1) $$K_t = \alpha' Y_{t-1} + \beta' \left(\frac{P_{t-1}}{K_{t-1}}\right) Y_{t-1}$$

(2.2) $$I_t = K_{t+1} - K_t = (Y_t - Y_{t-1})\left(\alpha' + \beta' \frac{P_{t-1}}{K_{t-1}}\right) + \beta' \left(\frac{P_t}{K_t} - \frac{P_{t-1}}{K_{t-1}}\right) Y_t$$

where $$\alpha' > 0, \quad \beta' > 0$$

(3) *Technical Progress Function*

$$\frac{Y_{t+1} - Y_t}{Y_t} = \alpha'' + \beta'' \frac{I_t}{K_t}$$

where $\alpha'' > 0$ and $1 > \beta'' > 0$

[1] Income is assumed to be divided into two categories, wages and profits, where the wage category comprises salaries as well as the earnings of manual labour; profits comprise not only entrepreneurial incomes but also incomes accruing to property generally.

Equation (1) shows the community's savings as consisting of a proportion α of aggregate profits (P_t) and a proportion β of wages ($Y_t - P_t$). Equation (2.1) shows that the stock of capital at time t (and which is assumed to be equal to the *desired* stock of capital at time $t - 1$) is a coefficient α' of the output of the previous period (Y_{t-1}) and a coefficient β' of the rate of profit on capital of the previous period, multiplied by the output of the previous period. Equation (2.2), derived from (2.1) by difference equation, shows that investment in period t, (I_t), assumed to correspond to the difference between desired and actual capital at t, is equal to the increment in output over the previous period ($Y_t - Y_{t-1}$) multiplied by the relationship between desired capital and output in the previous period $\left(\frac{K_t}{Y_{t-1}}\right)$[1] plus a coefficient β' of the change in the rate of profit over that period, multiplied by the output of the current period. Equation (2.2) thus implies that, expressed as a proportion of the existing stock of capital, K_t, the investment of period t is equal to the expected rate of growth of turnover (which in turn is assumed to be equal to the actual rate of growth in turnover for the previous period) if the rate of profit on capital is constant; and it is greater (or smaller) than this if the rate of profit on capital is rising (or falling). Equation (3) shows the rate of growth of labour productivity (and income) as an increasing function of the rate of net investment expressed as a proportion of the stock of capital—*i.e.*, of the (proportionate) rate of growth of the capital stock.

Starting from an arbitrary point of time, $t = 1$, we can regard the existing capital stock, K_1 as a datum, a heritage of the past. We can also take as given the income Y_1 which the fully employed labour force produces with the aid of the capital equipment K_1 and the income and capital in the previous period Y_0 and K_0.[2] Assuming that the capital stock K_1 satisfies the condition

$$\frac{K_1}{Y_0} = \alpha' + \beta' \frac{P_0}{K_0} \tag{2.1.2}$$

and treating, for the moment, K_1 as well as K_0 and Y_0 as a datum, equation (2.2) can be written in the form

$$\frac{I_1}{Y_1} = \frac{Y_1 - Y_0}{Y_0} \cdot \frac{K_1}{Y_1} + \beta' \left(\frac{P_1}{K_1} - \frac{P_0}{K_0}\right) \tag{2.3}$$

$\left(\text{bearing in mind that } \frac{Y_1 - Y_0}{Y_1} \cdot \frac{K_1}{Y_0} = \frac{Y_1 - Y_0}{Y_0} \cdot \frac{K_1}{Y_1}\right)$, which means that the rate of investment in period 1, as a proportion of the income of that

[1] Since it is implicit in equation (2.1) that

$$\alpha' + \beta' \frac{P_{t-1}}{K_{t-1}} = \frac{K_t}{Y_{t-1}}$$

[2] Only three of these magnitudes are independent, since they must satisfy the technical equation (3) above with $t = 0$.

period, equals the rate of growth of income over the previous period multiplied by the capital/output ratio of the current period, plus a term depending on the change of the rate of profit over the previous period. Equation (2.3) can in turn be written in the form

$$(2.4) \qquad \frac{I_1}{Y_1} = \left\{\frac{Y_1 - Y_0}{Y_0}\frac{K_1}{Y_1} - \beta'\frac{P_0}{K_0}\right\} + \beta'\frac{Y_1}{K_1}\cdot\frac{P_1}{Y_1}$$

while equation (1) can be written in the form

$$(1.2) \qquad \frac{S_1}{Y_1} = \alpha\frac{P_1}{Y_1} + \beta\frac{Y_1 - P_1}{Y_1} = \beta + (\alpha - \beta)\frac{P_1}{Y_1}$$

These two equations, (1.2) and (2.4) then determine both the distribution of income (between profits and wages) and the proportion of income saved

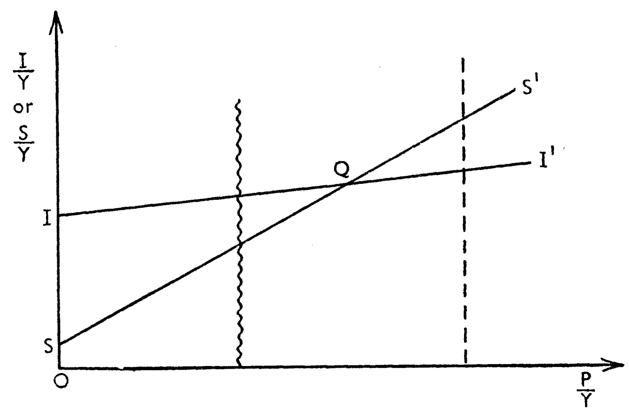

FIG. 2

and invested at $t = 1$. For the level of profits has to be such as to induce a rate of investment that is just equal to the rate of savings forthcoming at that particular distribution of income. This mechanism is illustrated in Fig. 2, where profits as a ratio of income $\left(\frac{P}{Y}\right)$ are measured horizontally and savings and investment as a ratio of income $\left(\frac{S}{Y} \text{ and } \frac{I}{Y}\right)$ vertically. The line SS' represents our equation (1.2) and II' our equation (2.4).[1] The point of intersection Q indicates the short-period equilibrium level of profits and of investment as a proportion of income. If profits are a lower proportion of income the investment plans (although lower than the equilibrium level) will tend to exceed the available savings; prices will rise in relation to costs, until the discrepancy is eliminated through the consequential rise in profits. The equilibrium will be stable if the slope of the SS'

[1] The starting point of SS' on the vertical axis represents β and its slope $\alpha-\beta$. The starting point of II' on the vertical axis depends on the proportional change in income in the previous period and on the rate of profit on capital in the previous period while the slope of II' is $\beta'\frac{Y_1}{K_1}$.

curve exceeds the slope of the II' curve, which implies, that the coefficients of the first two equations satisfy the condition

$$\alpha - \beta > \beta' \frac{Y_t}{K_t}$$

We shall assume that this is so.[1]

The working of the model is subject to two further restrictions, which were explained in an earlier article,[2] and which can be expressed as follows:

$$(4) \quad P_t \leqslant Y_t - W_{\min}$$

$$(5) \quad \frac{P_t}{Y_t} \geqslant m$$

The first of these restrictions (4) means that the profits determined by equations (1) and (2.2) should not be greater than the surplus available after the labour force has been paid a subsistence wage-bill. If this condition were not satisfied investment would be less than that indicated by equation (2.2) and would be determined by the savings available according to equation (1), when profits are equal to the surplus over subsistence wages. (In Fig. 2 the vertical dotted line represents the maximum permissible level of $\frac{P}{Y}$, and this is obtained by deducting the subsistence wage-bill from the full-employment income, determined by this condition. If the dotted line were to fall to the left of Q, as in Fig. 3, the short-period equilibrium would be represented not by Q, but by R. It will be evident that the position of the dotted line in relation to Q depends on the productivity of labour, which in turn depends in our model on the capital stock. In a progressive economy with a growing capital and output per head the dotted line will move steadily to the right, so that *sooner or later* it will pass Q and equation (2) will become operative as in Fig. 2.[3])

The second of these restrictions (equation (5)) means that the profits resulting from equations (1) and (2.2) are higher than the minimum required to secure a margin of profit over turnover below which entrepreneurs would not reduce prices, irrespective of the state of demand. (This minimum

[1] The assumption of stability in the savings–investment equilibrium under full employment has quite different implications from the corresponding assumption in a situation of under-employment equilibrium where the short-period output is regarded as variable. In the latter case the stability conditions depend on the effect of the change of income on the proportion of investment to income whereas in the former case only on the corresponding effect of a change in the *distribution* of income. In terms of our equation, a necessary (though not a sufficient) condition for short-period stability in an under-employment equilibrium—irrespective of whether equation (1) holds or not, and provided only that the marginal propensity to consume is higher than zero—is that $K_t < Y_t$ (*i.e.*, the accelerator coefficient in the first term of our second equation is less than unity). In a full-employment model the stability of our equilibrium is not dependent at all on the accelerator coefficient, but only on the change of that coefficient induced by a change in the rate of profit associated with a change in the distribution of income.

[2] Cf. " Alternative Theories of Distribution," *op. cit.*, pp. 97–8.

[3] Cf. also pp. 618–21 below.

margin of profit is that corresponding to the " degree of monopoly," or the traditional profit margin required to cover full costs, etc.) If this condition were not satisfied, full-employment savings indicated by equation (1) would exceed investment (since prices and profits would not fall sufficiently to secure the equality of savings and investment), so that income and employment would be reduced *below* the full-employment level to the point where

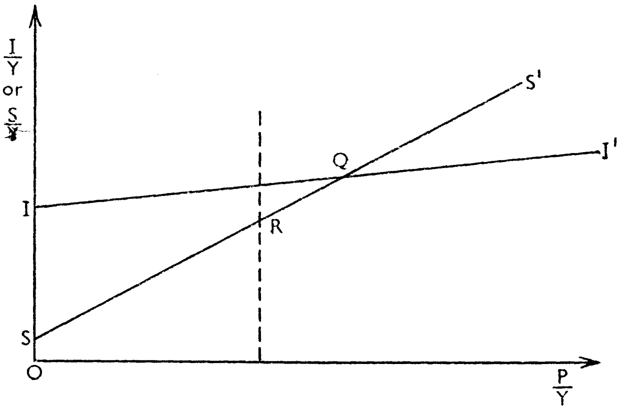

Fig. 3

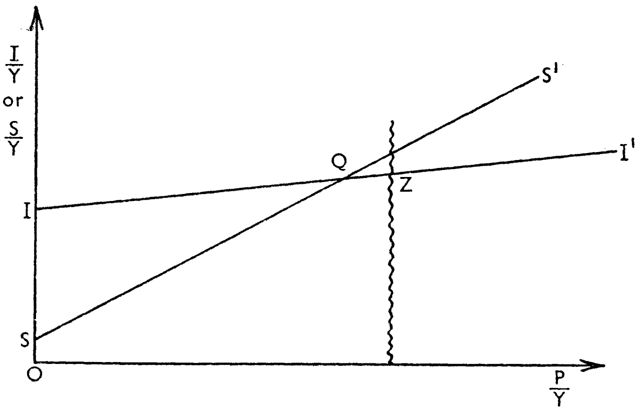

Fig. 4

the savings generated by that income are no more than sufficient to finance investment. (In Fig. 2 the waved line represents this minimum level of profits. If this minimum were to fall to the right of Q equilibrium would not be at Q, but at Z, as shown in Fig. 4, while income will fall below Y_0 to the point where the savings–income ratio is reduced to the level indicated by Z.)

Our model thus supposes: (i) that the wages $(Y_0 - P_0)$ resulting from equations (1) and (2.2) are higher than the minimum, set by the supply-price of labour; (ii) that the profits resulting from these same equations are higher than the minimum required to satisfy entrepreneurs. The absence of the first condition leads to the Marxian type of model, where profits are determined by the surplus over subsistence wages, and investment is governed by the size of that surplus; the absence of the second condition leads to a Keynesian model of under-employment equilibrium, which, as was indicated above,[1] is inconsistent with the long-run equilibrium of a growing economy. Our model thus relates to a capitalist economy which is sufficiently highly developed for wages to be above subsistence level and sufficiently competitive at the same time to generate adequate demand to secure full employment.

Assuming these conditions are satisfied, our technical progress function (equation (3) above) indicates the growth of income and capital from $t = 1$ onwards, and the gradual movement of the economy from a short-period equilibrium to a long-period equilibrium of steady growth. This is illustrated in Fig. 5, where the proportionate growth of capital is measured

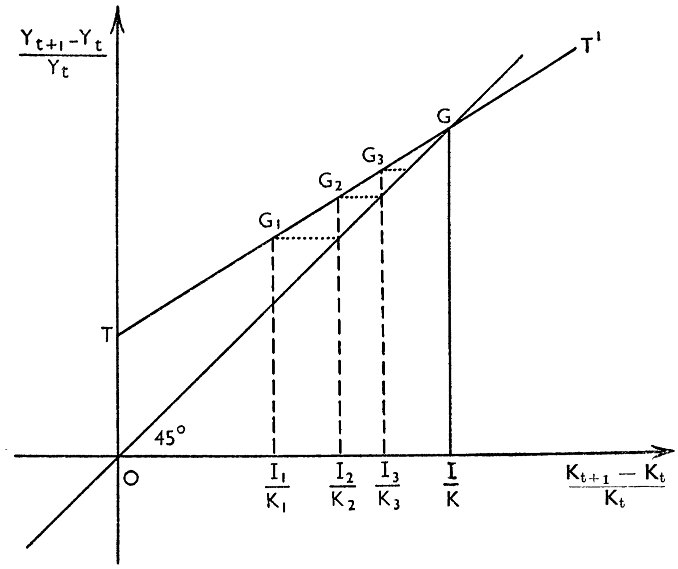

FIG. 5

horizontally and the proportionate growth of income vertically.[2] Suppose that the initial rate of investment, as determined by equations (1) and (2.3), at $t = 1$, $\frac{I_1}{K_1}$ is to the left of $\frac{I}{K}$. This implies that the growth of output, g, in successive units of time will be greater than the growth of capital, $\frac{I_1}{K_1}$,[3] and, in accordance with our equation (2.3), and apart from any changes in the rate of profit on capital, the rate of investment will be stepped up in the

[1] Cf. p. 594 above.

[2] Fig. 5 is identical with Fig. 1 except that in accordance with equation (3), the technical progress function TT' is here represented by a straight line.

[3] We shall denote g_1, g_2, etc., the rates of growth in income corresponding to the points G_1, G_2, etc., in the diagram.

subsequent period so as to make $\frac{I_2}{K_2}$ equal to g_1, which in turn will raise the growth in income in the second period to g_2. By similar reasoning, the growth of output in the third period will rise to g_3 and so on, until G is reached, at which the rates of growth of income and capital are equal.

The indirect effects through changes in the rates of profit on capital will reinforce this process. For since $\frac{Y_t}{K_t}$ is increasing so long as G_t is to the left of G, $\frac{P_t}{K_t}$ will be increasing so long as $\frac{P_t}{Y_t}$ is not decreasing; while, in accordance with equation (1), $\frac{P_t}{Y_t}$ will be non-decreasing provided $\frac{I_t}{Y_t}$ is not decreasing. The change in $\frac{I_t}{Y_t}$ attributable to the first term of (the right-hand side of) equation (2.2) will be positive with any movement of G_t towards G, for the rise in g_t will more than offset the fall in the capital/output ratio, $\frac{K_t}{Y_t}$. It follows that any associated change in the rate of profit on capital $\frac{P_t}{K_t}$ will make the rate of increase in investment even greater. (This means that in Fig. 2 above the point of intersection Q must be moving towards the right, which implies also that the curve II' must be shifting upwards.[1])

[1] A more rigorous proof of the proposition that equation (2) represents a curve in the $\left(\frac{I_t}{Y_t}, \frac{P_t}{Y_t}\right)$ plane which moves upwards when the percentage growth in income exceeds the percentage growth in capital can be given as follows.

Since, according to equation (2.1) and (2.2)

$$K_{t+1} = \left\{\alpha' + \beta' \frac{P_t}{K_t}\right\} Y_t$$

Equation (2.2) can be written in the form

$$I_t = K_{t+1} - K_t = \alpha' Y_t - K_t + Y_t \left\{\beta' \frac{Y_t}{K_t}\right\} \frac{P}{Y}$$

hence

$$\frac{I_t}{Y_t} = \left\{\alpha' - \frac{K_t}{Y_t}\right\} + \left\{\beta' \frac{Y_t}{K_t}\right\} \frac{P_t}{Y_t}$$

This curve must evidently shift upwards with any increase in $\frac{Y_t}{K_t}$, for any fixed value of $\frac{P_t}{Y_t}$.

On the other hand, if instead of equation (2.2) we had chosen an investment function of the form

$$\frac{I_t}{K_t} = \alpha' + \beta' \frac{P_t}{K_t} \quad (\text{where } \alpha' > 0),$$

which makes the proportionate rate of growth of capital depend simply on the rate of profit on capital and without regard to the rate of change of income, the relationship of investment to income will be

$$\frac{I_t}{Y_t} = \alpha' \frac{K_t}{Y_t} + \beta' \frac{P_t}{Y_t}$$

In that case if $\frac{\Delta Y_t}{Y_t} > \frac{I_t}{K_t}$, so that $\frac{K_t}{Y_t}$ is falling, this curve $\frac{I_t}{Y_t}$ in the $\left(\frac{I_t}{Y_t}, \frac{P_t}{Y_t}\right)$ plane will be falling. An investment function of this form therefore, whilst it satisfies the conditions of stability of short-period equilibrium, makes the position of long-period equilibrium at G unstable, since the point G_t in Fig. 5 would be moving away from instead of towards G.

An exactly similar proof could be adduced to show that if G_t were to lie to the right of G, then it must be moving to the left towards G. Long-run equilibrium must therefore be at G, where the rates of growth of income and capital are equal.

It follows from our equations that the long-run equilibrium rate of growth of income and capital is independent of the value of the coefficients of equations (1) and (2.3) (the savings and investment functions), and depends only on the coefficients in equation (3), the technical progress function. It is given by

$$(6) \qquad G = \frac{\alpha''}{1 - \beta''}$$

which is the equilibrium rate of growth in productivity, *i.e.*, that particular rate of growth of productivity which makes the (percentage) rates of growth of capital and income equal, and which (under the hypothesis of a constant population) is itself equal to them both.

Putting

$$\frac{\alpha''}{1 - \beta''} = \gamma'',$$

the equilibrium ratio of investment to income, the equilibrium share of profits to income and the equilibrium rate of profit on capital can all be derived with the aid of equations (1) and (2.3) as follows:

$$(7.1) \quad \frac{I}{Y} = \gamma'' \frac{K}{Y}$$

Since from equation (1.2)

$$\frac{S}{Y} = \alpha \frac{P}{Y} + \beta \left(1 - \frac{P}{Y}\right)$$

$$\therefore \quad (8.1) \quad \frac{P}{Y} = \frac{\gamma'' \frac{K}{Y} - \beta}{\alpha - \beta}$$

$$(9.1) \quad \frac{P}{K} = \frac{\gamma'' - \beta \frac{Y}{K}}{\alpha - \beta}$$

The family resemblance between our set of equations and the Harrod–Domar formula will be evident. Equation (7.1), together with equation (1.2), is a variant of Mr. Harrod's " warranted rate of growth," with the important differences: (i) that the assumption of given savings propensities (for profit-receivers and wage-earners) does not define a unique warranted rate, but is consistent with any number of warranted rates, depending on the distribution of income, which determines the average propensity to save for the community as a whole, the latter in turn being dependent on the ratio of investment to income; (ii) the rate of growth of the system is not determined by the savings function, but by equation (6), which is, in effect, a variant of Mr. Harrod's " natural rate " (under the hypothesis of a constant population) except that the rate of increase in productivity due to technical progress is not treated as a constant here, but as a variable, and our γ'' is that particular rate of increase of productivity which makes the latter equal to the rate of increase in capital per head. In fact, the implications of our model in terms of Mr. Harrod's terminology could be summed up by saying that the system tends towards an equilibrium rate of growth at which the " natural " and the " warranted " rates are equal, since any divergence between the two will set up forces tending to eliminate the difference; and these forces act partly through an adjustment of the " natural " rate, and partly through an adjustment of the " warranted " rate.

So far the expression $\frac{K}{Y}$, though an outcome of the equilibrating process, has not been eliminated from our formulæ. To do so, we must return to our desired capital function (2.1), which, owing to the substitution introduced in (2.1.2), has not so far been made use of. Dividing (2.1) with Y_t, we obtain

$$(2.1.3) \qquad \frac{K_t}{Y_t} = \alpha' \frac{Y_{t-1}}{Y_t} + \beta' \left(\frac{P_{t-1}}{K_{t-1}}\right) \frac{Y_{t-1}}{Y_t}$$

Bearing in mind that in long-period equilibrium

$$\frac{P_{t-1}}{K_{t-1}} = \frac{P_t}{K_t} = \frac{P}{K}, \quad \text{and} \quad \frac{Y_{t-1}}{Y_t} = \frac{1}{1+\gamma''}$$

and putting

$$\frac{K_t}{Y_t} = \frac{K}{Y} = x$$

we obtain the expression

$$(2.1.4) \qquad x = \frac{1}{1+\gamma''}\left(\alpha' + \beta' \frac{P}{K}\right)$$

Further, since in long run equilibrium

$$\frac{S_t}{K_t} = \frac{S}{K} = \gamma''$$

equation (1.2), divided by K_t, can be written in the form

$$(1.3) \qquad \gamma'' = \frac{\beta}{x} + (\alpha - \beta)\frac{P}{K}$$

Hence

$$\frac{P}{K} = \frac{1}{\alpha - \beta}\left\{\gamma'' - \frac{\beta}{x}\right\}$$

and

$$x = \frac{1}{1+\gamma''}\left\{\alpha' + \frac{\beta'}{\alpha - \beta}\left(\gamma'' - \frac{\beta}{x}\right)\right\}$$

which can be re-written in the form

$$(\alpha - \beta)(1 + \gamma'')x^2 = \{(\alpha - \beta)\alpha' + \beta'\gamma''\}x - \beta\beta'$$

Hence

$$(10.1) \qquad Ax^2 - Bx + C = 0$$

where $A = (\alpha - \beta)(1 + \gamma'')$, $B = (\alpha - \beta)\alpha' + \beta'\gamma''$, and $C = \beta\beta'$

Thus the derivation of the capital/output ratio, $\frac{K}{Y}$, involves a quadratic equation which has two positive roots in $\frac{K}{Y}$, of which normally the greater of the two only would be relevant.[1]

The complication of quadratic solutions is avoided, however (and the formulæ greatly simplified), if we assumed that all savings come out of profits, so that $\beta = 0$. (Since β is in any case likely to be small, the difference introduced by this cannot quantitatively be very significant.) In that case the equilibrium value of the capital/output ratio, derived from (2.1.4) and (1.3), is given by the expression

$$(10.2) \qquad \frac{K}{Y} = \frac{\alpha\alpha' + \beta'\gamma''}{\alpha(1 + \gamma'')}$$

while the equilibrium values of the other variables reduce to the formulæ

$$(7.2) \qquad \frac{I}{Y} = \frac{\alpha\alpha'\gamma'' + \beta'(\gamma'')^2}{\alpha(1 + \gamma'')}$$

$$(8.2) \qquad \frac{P}{Y} = \frac{\alpha\alpha'\gamma'' + \beta'(\gamma'')^2}{(\alpha)^2(1 + \gamma'')}$$

$$(9.2) \qquad \frac{P}{K} = \frac{\gamma''}{\alpha}$$

It is interesting to note that the rate of profit on capital depends on the rate of growth, γ'' (and thus ultimately on the coefficients of the technical progress function, α'', and β'' which determine this rate), and on the savings coefficients of profits, α. On the other hand, the investment coefficient and the share of profits in income, just as the capital/output ratio, depend on the coefficients of the investment function, α' and β', as well as on α and α'' and β''.

The result shown by equation (9.2) that the rate of return on capital depends only on the rate of economic growth and the division of capitalists' income between consumption and saving, and is independent of everything else (such as the factors determining the share of profits in income and the capital/output ratio) may sound highly paradoxical at first. Yet a little reflection shows that it must evidently be correct. For if the income accruing to capital were all devoted to accumulation (when that income is

[1] Since on any reasonable values o the coefficients, the larger root will be greater than unity and the smaller root less than unity when Y is thought of as annual income, and $\frac{K}{Y}$ is likely to be appreciably larger than 1. (The same holds, of course, with appropriate adjustments in the values of the coefficients, when Y is thought of in terms of some different period.)

the sole source of savings) the rate of profit on capital would evidently be identical with the rate of growth of the capital stock; if capital and output grow at the same rate this rate must be identical with the rate of growth of the economy. If the owners of capital do not save all their income but consume part of it (but profits remain the only source of savings) the rate of profit must exceed the rate of accumulation by the ratio of capitalists' consumption to savings.[1]

(b) *Expanding Population*

According to the Malthusian theory, the rate of population increase is a function of the rate of increase of the " means of subsistence " (which we can assume to be equivalent to the rate of increase in total production.) This doctrine is clearly subject to certain limitations. For any given fertility rate (or gross reproduction rate) in a community the percentage rate of growth in population cannot exceed a certain maximum, however fast real income is rising; and it may be supposed that the rate of population growth will rise only moderately as a function of the rate of growth in income over some interval of the latter before that maximum is reached. The dependence of population growth on the growth of income is best represented therefore in the terms of a curve which is convex upwards, as in Fig. 6 (measuring the proportionate rate of growth of population vertically and that of income horizontally) and whose slope nearly equals unity when the rate of growth of income is relatively low, and which becomes virtually horizontal when the rate of growth of income exceeds a certain critical value.[2] In terms of a linear equation, this relationship can be approximated by two straight lines indicated by the dotted lines in Fig. 6, which can be algebraically expressed as follows. Writing l_t, g_t for the (percentage) rates of growth of population and income, and λ for the maximum rate of growth of population,

$$(11) \qquad \begin{aligned} l_t &= g_t \ (g_t \leqslant \lambda) \\ l_t &= \lambda \ (g_t > \lambda). \end{aligned}$$

[1] As (9.1) shows, when profits are not the sole source of savings, the rate of return on capital will also depend on wage-earners' savings, though in no simple fashion, since β (the proportion of wages saved) appears both in the numerator and the denominator of the expression. It follows, however, from (8.1) that $\gamma'' < \alpha \frac{Y}{K}$ when $\frac{P}{Y} < 1$, hence a rise in β must lower the rate of profit on capital (and the share of profits in income). This is the basis of the (seemingly paradoxical) assertion that whereas the capitalists (as a group) can raise their share in the national income by spending *more*, wage-earners as a group can increase their share only by spending *less*.

[2] The maximum rate of population growth is partly a matter of fertility rates (*i.e.*, the gross reproduction rate) and partly of medical knowledge which determines the rate of survival (in particular, the infant-mortality rates) at a given standard of living. The fall in fertility rates in the advanced countries during the last half-century or more lowered the position of their Malthusian population curves considerably. On the other hand, there is a good deal of evidence tending to suggest that the considerable acceleration in the rates of population growth of the under-developed areas which occurred in the last half-century or so was due, in the main, not to an acceleration in the rate of growth of income but to a rise in the survival rate (due to the medical improvements which reduced the incidence of epidemics, etc.) and was attended in some cases by an appreciable fall in the standard of living of the population.

Assuming to start with that the rate of population growth is λ (*i.e.*, $g_t > \lambda$), $\frac{I_t}{K_t}$ in equation (3) is replaced by $\frac{I_t}{K_t} - \lambda$, and $\frac{Y_{t+1} - Y_t}{Y_t}$ by $\left(\frac{Y_{t+1} - Y_t}{Y} - \lambda\right)$. Hence the long-run equilibrium rate of growth of both capital and income becomes

$$G = \gamma'' + \lambda \tag{6.2}$$

The long-run equilibrium values of other ratios are then obtained by substituting $(\gamma'' + \lambda)$ for γ'' in equations (7)–(10).

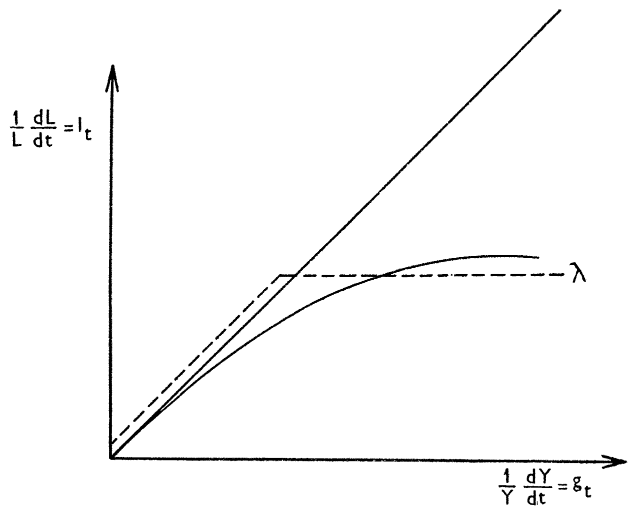

FIG. 6

If to start with, $g_t < \lambda$ (and hence $l_t < \lambda$) the rates of growth of income and population will continually accelerate until the latter approaches λ. In long-run equilibrium population must therefore grow at its maximum rate—*i.e.*, at that indicated by the horizontal section of the curve in Fig. 6.

The above assumes that the shape and position of the technical progress function—*i.e.*, the coefficients α'' and β'' in equation (3) above, and hence γ''—remain unaffected by population changes. This implies in economic terms that there are constant returns to scale to equi-proportionate increases in labour and capital; in other words, that an increase in numbers, given the amount of capital per head, leaves output per head unaffected. This assumption may be valid enough in the case of a young and relatively under-populated country [1]; in the case of over-populated countries, however, the

[1] In general, the density of population in any given area will in itself be conditioned by the availability of natural resources, which means that the density will normally be sufficiently great for the stage of diminishing returns to have been reached. When, however, a new population possessing radically different techniques invades a territory (as in the case of the white settlers in America and other areas in the New World) the point of optimum density, corresponding to the new techniques, may be so radically different that there may occur a manifold increase in population density without encountering diminishing returns.

scarcity of land will cause diminishing returns, which means that, with given techniques and capital per head, an increase in numbers will cause a fall in productivity.[1] Given the rate of the flow of new ideas, the curve denoting our technical progress function will be lowered by an extent depending on the rate of increase in population. It is then possible that instead of the curve intersecting the income-axis in the positive quadrant, as shown in Fig. 1 above, it will cut the capital-axis positively, as in Fig. 7 [2]—which means that it will require a certain percentage growth in capital per head (C_t) even to maintain output per head (O_t) at a constant level. It will be evident that the maintenance of an equilibrium of growth is much more precarious in this case; instead of a single point of intersection of the curve

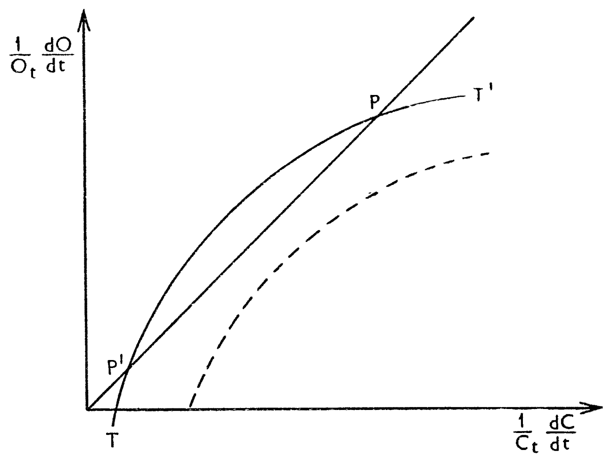

FIG. 7

with the diagonal line, we have two points of intersection, P' and P, of which only the latter is stable, while the former is unstable; if the economy happens to be in a position (or come to a position) which is to the left of P' the rate of growth of income and capital will steadily diminish until—after an intervening period of falling output per head—the growth of capital and income come to a complete standstill. It is even possible in this case that the position of

[1] It is assumed here that diminishing returns due to the scarcity of land are attendant upon an increase in the working population, rather than an increase in the volume of production as such. It is possible, of course, that the scarcity of natural resources would put increasing obstacles to the expansion of output, even when population is constant—in which case, given the rate of invention and innovation, the growth of productivity at any given rate of investment will be slowed down, and our TT' curve will gradually shift downwards over successive periods. Since, however, diminishing returns due to the scarcity of land are peculiarly associated with food supplies, while food requirements vary with the size of the population far more than with income per head, it is more appropriate to treat the problem of diminishing returns in the classical manner and associate it with the " widening " of capital due to an increase in numbers, rather than as restricting the scope of increasing productivity through " deepening."

[2] Since the assumption of a linear function is no longer adequate in this case for exhibiting the various possibilities inherent in the situation, we return to the non-linear mode of representation which was originally employed in Fig. 1.

the TT' curve should be below the diagonal throughout its length (as in the dotted line in Fig. 7), in which case no long-period equilibrium is attainable short of complete stagnation.

Whether an expanding population will be consistent with an equilibrium of growth or not will thus primarily depend on the relative magnitude of two factors: (i) the maximum rate of population increase λ, and (ii) the rate of technical progress, which causes a certain percentage increase in productivity, α'' in equation (3) above, when both population and capital per head are held constant. Since diminishing returns cannot cause the output of a larger working population to be smaller than that of a smaller population, the growth of population cannot lower the position of the TT' curve by more than the rate of population growth itself, so that *if*

$$\alpha'' > \lambda$$

the technical progress function must continue to cut the vertical (income) axis positively, and the possibility of a stable equilibrium of growth will be assured. The long-run equilibrium rate of growth will still be given by the formula

$$G = \gamma'' + \lambda \tag{6.2}$$

bearing in mind, however, that the value of γ'' is not here independently given, but is itself influenced by λ.[1] But when the value of λ is relatively large, and the forces making for technical progress are weak, the formula (6.2) may no longer apply because of the inability of income to grow at a steady rate of λ or above. For example, if we suppose that, as a result of diminishing returns and rapid population growth (made possible by the high value of λ), α'' in equation (3) became negative—which implies that in Fig. 5 the point of intersection G with the diagonal had moved down into the negative quadrant, making γ'' negative—the growth of population would exceed the growth of production when the population grew at the rate λ. In this situation an equilibrium rate of growth is conceivable only when population and income grow at the same rate, and this can be attained only at that particular rate of population growth (lower than λ) which makes γ'' equal to zero. Thus, if we denote by $L(\gamma)$ the rate of population growth which causes γ'' to take the value γ, then in place of equation (6.2) we have

$$(12) \qquad \begin{aligned} l_t &\longrightarrow L(o) \\ g_t &\longrightarrow l_t \\ \therefore \quad g_t &\longrightarrow L(o) \end{aligned}$$

[1] The available data for the more advanced capitalist economies over the last century suggest that the value of γ'' varied between 2 and 4% per annum, for the different countries, which is consistent with a value of β'' of, say, 0·5, and for α'' of 0·01–0·02 or 1–2%. According to Professor Kuznets' investigations (cf. "The Quantitative Aspects of the Economic Growth of Nations," *Economic Development and Cultural Change*, Vol. V, No. 1, October 1956, p. 42), the value of γ'' in the more advanced economies remained relatively unaffected by changes in the rate of population growth. In the under-developed areas, according to the evidence of the same source, the value of γ'' must have been very much lower.

In other words, for economies which are capable of only slow technical progress, and whose potential rate of population growth is relatively large, and which are subject to diminishing returns, the long-run equilibrium rate of growth and income (and capital) is determined by a different set of conditions. It has to be that rate of population growth which allows output per head and capital per head to remain constant over time. (Income and capital per head must be low enough to restrict the rate of population growth to that rate; and the higher the degree of medical knowledge which determines infant mortality, the lower this constant level of income per head will have to be.) This again is a stable position, since there is only one particular rate of population growth which enables the rate of growth of income to be equal to it; at any lesser rate productivity per head will rise, and the growth of income will exceed the growth of population (causing the latter to increase and income per head to cease rising); at any higher rate, productivity will fall and the growth of income will fall short of the growth in population (causing the latter to contract and income per head to cease falling). Long-run growth with a rising standard of living necessarily presupposes that there is some check to the rate of population growth which operates before it reaches the maximum attainable rate of growth of the national income.

THE TWO STAGES OF CAPITALISM

The historical emergence of capitalist enterprise involved a tremendous increase in the " technical dynamism " of the economic system. The most important characteristic of capitalist business enterprise is the continuous change and improvement in the methods of production, as against the relatively unchanging techniques of peasant cultivation and artisan production. In terms of our model, the growth of the capitalist sector in the economy involved a dramatic rise in technical progress function, and hence in the equilibrium rate of growth of productivity, γ''—the increase in savings, investments, both as a proportion of income and of capital, and the great acceleration in the rates of population growth, were consequences of this, and not its initiating causes.[1]

[1] In the same way the important differences in long-period growth rates between different capitalist economies—which manifested themselves, *e.g.*, in the fivefold increase in income per head in the United States in the last 100 years as against a near threefold increase in Britain over the same period—can only be ascribed to the various social factors which cause differences in the degree of " technical dynamism "—in the speed of adaptation to new techniques—rather than to differences in savings propensities, in national environment, etc. Again, the causes of the relatively low rates of progress in the under-developed areas of the world are mainly to be sought in the social and institutional factors which impeded the spread of " technical dynamism," particularly in the agricultural field, and thereby inhibited progress in those sectors of the economy also in which capitalist enterprise could establish itself and where the sociological obstacles to continued technical change were removed. Thus the absence of a progressive agriculture has been the most important factor inhibiting industrialisation in the under-developed areas. (This latter problem is treated in more detail in another paper by the author, " Characteristics of Economic Development," published in *Atti del Congresso Internazionale di Studio sul Problema delle Area Arretrate* (Milan: Giuffre 1955).)

In the early stage of capitalist development the growth in productivity was not attended by a rise in the standard of living of the working classes. The stationary trend in real wages in Britain despite considerable improvement in production per head during the first half of the nineteenth century was the feature of capitalist evolution which so strongly impressed Marx and forms one of the main themes of Volume I of *Das Kapital*. The same has been true of other capitalist countries: in the case of Japan, for instance, real wages increased very little between 1878 and 1917, despite a one-and-a-half-fold increase in real income per head over the period.[1]

This suggests that in the first stage of capitalist evolution, productivity, though rising, is not large enough to allow for a surplus over the subsistence wage which permits the rate of investment to attain the level indicated by equation (2.2); in other words the profits which would result from equations (1) and (2.2) are inconsistent with the restriction indicated by equation (4).

Since equation (2.2) will be replaced by

$$P_t = Y_t - W_{\text{min.}} \tag{4a}$$

this combined with (1) yields

$$S_t = (\alpha - \beta)\, P_t + \beta Y_t$$

so that

$$S_t = I_t = \alpha Y_t - (\alpha - \beta)\, W_{\text{min.}} \tag{13}$$

So long as (13) holds $\frac{I_t}{Y_t}$ will be steadily rising with the increase in the productivity of labour so that if, initially, $\frac{Y_t}{K_t}$ is rising (*i.e.*, the situation is to the left of G in Fig. 5) $\frac{I_t}{K_t}$ will be rising also. This movement will not be brought to a halt by approaching G, for while the (current) rate of growth of capital will approach the (current) rate of growth of income (as the rate of investment approaches that corresponding to G), there will be a backlog of investment from previous periods due to the growing divergence between actual capital and desired capital (since the rate of investment at each period was *ex hypothesi* insufficient to bring actual capital into line with desired capital). Hence the increase in the rate of capital accumulation will pass the point G, and the economy will settle down to a steady decrease of $\frac{Y_t}{K_t}$. In this first stage of capitalism therefore the capital/output ratio (at any rate after some initial period) will show a steady increase, in accordance with both the Marxian and the neo-classical models. Since, however, the share of profit in income will also increase continuously, the rise in the

[1] Cf. Mataji Umemura. "Real Wages of Industrial Workers" in the volume *An Analysis of the Japanese Economy*, edited by S. Tsuru and K. Ohkava (Tokyo, 1953). In the period 1918–42, on the other hand, the rise in real wages fully kept pace with the growth in output per head.

capital/output ratio will not necessarily imply a falling rate of profit on capital, and may be consistent with a rising rate.

This first stage of capitalism, however, must sooner or later be brought to an end when the capital stock attains the level of " desired capital," indicated by equation (2.1). From that point onwards the rate of investment will no longer be governed by equation (13), but by equation (2.2) and the reaction-mechanism of the system becomes a totally different one. Profits are no longer determined in the Marxian manner, as the surplus of production over subsistence wages; on the contrary, the share of wages becomes a residual, equalling the difference between production and the share of profits as determined in a " Keynesian " manner, by the propensities

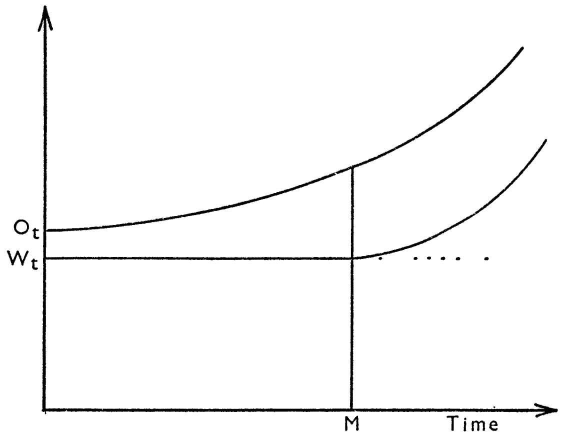

Fig. 8

to invest and to save. From then onwards, and assuming that the parameters in our equations (1)–(3) remain constant, real wages will rise automatically at the same rate as the productivity of labour, so that distributive shares remain constant through time; and since the system will tend to settle down to an equilibrium where the rate of growth of capital is equal to the rate of growth in income, the capital/output ratio, and the rate of profit in capital, will also tend to remain constant over time.

The process of changing-over from Stage I to Stage II is illustrated in Fig. 8 (measuring time horizontally, and productivity, O_t, and wages, W_t, vertically). The dividing line M is at the point of time when the stock of capital attains the " desired " level, indicated by equation (2.1) and the share of profits $(O_t - W_t)$ yields savings that are sufficient to finance a rate of investment corresponding to equation (2.2). Once this stage is reached, any further increase in the " surplus " will not be fully absorbed in increasing investment and/or capitalist consumption; a growing share of the increase in the " surplus " will accrue to labour automatically, through the constellation of prices in relation to wages.

This second and more cheerful stage of capitalism in which production and employment continue to grow, and real wages are steadily rising with the growth in production, was quite unforeseen by Marx. Marxist economists would probably argue that its emergence is prevented by the growth of " monopoly-capitalism," for not only the productivity of labour, but also the degree of concentration of production can be expected to rise steadily with the progress of capitalism. This causes a steady weakening of the forces of competition, as a result of which the share of profit would go on rising beyond the point where it covers investment needs and the consumption of capitalists. Hence, on this argument, by the time the restriction implied in equation (4) is lifted the restriction implied by equation (5) should become operative, and that means that the system will cease to be capable of generating sufficient purchasing power to keep the mechanism of growth in operation.

The plain answer to this is that so far, at any rate, this has not happened. Though the growing concentration of production in the hands of giant firms proceeded in much the same way as Marx predicted, this was not attended by a corresponding growth in the share of profits.[1] On the contrary, all statistical indications suggest that the share of profits in income in leading capitalist economies such as the United States have shown a falling rather than a rising trend over recent decades, and is appreciably below the level of the late nineteenth century; and despite the extraordinary severity and duration of the depression of the 1930s, the problem of "realising surplus value " appears no more chronic to-day than it was in Marx's day.

Trend and Fluctuations

Our model is intended to be a " long-run " model—*i.e.*, a model exhibiting long-run tendencies operating in the economy—and as such it deliberately rules out all kinds of complicating features of the real world which have to be taken into account before the methods of reasoning and the conclusions can be applied to actual situations. In this paper we can do no more than indicate briefly some of these limitations and the influence they are likely to exert, particularly during shorter periods.

(1) One of these is that the theory of distribution underlying this model —which makes the share of profits in income entirely dependent on the ratio of investment to output, and the propensity to save out of profits and wages

[1] Moreover—and this is more significant—empirical investigations concerning the ratio of profit to turnover in different industries do not lend any support to the hypothesis that the differences in profit margins are to be explained by differences in the degree of concentration of production. Typically monopolistic industries, where output is largely controlled by a few firms, have in many cases lower profit margins than industries where the degree of concentration is small.

—is only acceptable as a " long-run " theory, since changes in these factors exert only a limited influence in the short period. As was indicated in my earlier paper,[1] in the short period profit margins are likely to be inflexible, in both an upward and a downward direction, around their customary level —which means that they are largely historically determined. What is suggested here is that long-term investment requirements and saving propensities are the underlying factors which set the standard around which these customary levels are formed, and which are responsible for the gradual change of these levels in any particular economy, or for differences as between different economies. This means that in the short period: (i) when investment falls significantly *below* some " normal " level, profit margins will not fall sufficiently to set up a compensating increase in consumption; instead, total income and employment will be reduced, in accordance with the Keynesian multiplier theory; (ii) when investment demand rises significantly *above* some " normal " level profit margins will not rise sufficiently to allow a corresponding increase in real investment; instead, some kind of investment rationing will take place by the lengthening of order books, and/or a tight credit policy, etc., or simply by the rise in the prices of investment goods in relation to consumption goods.[2] The short-period rigidity of profit margins, due to entrepreneurial behaviour, will be reinforced also by another factor: the downward inflexibility of real wages around their customary or attained level. Though over a longer period the *share* of wages is flexible in both an upward and downward direction through *real* wages rising more or less than in proportion to the rise in productivity, in the short period an *absolute* cut in real wages is likely to entail a severe inflationary wage–price spiral; and hence an increase in investment which would entail such a cut is likely to be prevented, if by nothing else, by measures of monetary policy. The speed with which an increase in the proportion of current production devoted to investment can be brought about will therefore be limited by the rate of increase in productivity, as well as by other factors, such as the limited capacity of investment-goods industries.

(2) A second important qualification relates to the hypothesis of a constant flow of invention and innovation over time, which underlies our assumption of a technical-progress function with constant parameters. To the extent that technical progress consists of a great multitude of minor changes and improvements, one may rely on the operation of the law of large numbers

[1] Cf. " Alternative Theories of Distribution," *loc. cit.*, pp. 99–100.

[2] This is in any case part of the mechanism through which a rise in investment demand will cause in the long run a rise in the proportion of resources devoted to investment and a corresponding fall in the proportion devoted to consumption. In the short period, investment in real terms is limited by the capacity of the investment-goods industries; a rise in the demand for investment goods must therefore raise first their relative prices, and thereby the relative profitability of investment in these industries; this in turn will entail a gradual rise in the capacity of these industries, relatively to the capacity of the consumption-goods industries.

to ensure that the rate at which improvements are invented and introduced in the economy as a whole remains fairly steady. But in addition there are major innovations due to the discovery of basic new principles; these occur at irregular intervals, and their exploitation opens up vast new avenues of profitable investment. The effects of such major discoveries (such as the invention of electricity, the internal-combustion engine, etc.) are superimposed on the " normal " rate of progress due to minor improvements, and their effect is to raise the TT' curve above its " normal " level during the period of their initial exploitation (which may be spread over several decades). Hence we can expect periods of rising and falling technical progress: periods in which the growth of production and real income runs ahead of the growth of capital, to be followed by other periods when capital investment catches up and the stock of capital increases faster than income.

The effect of the short-period rigidities of profit margins referred to in the earlier paragraph is to slow down the rate at which capital investment is rising in response to an upward shift of the TT' curve; and also to slow down the rate at which actual investment is falling in response to a downward shift of the TT' curve. To a certain extent, therefore, these short-period rigidities act as a stabiliser on the economy; they make it capable of absorbing the shocks created by the unequal incidence of new discoveries without major unheavals—merely by varying the rates at which income and capital are growing.[1] However, these same short-period rigidities may cause " over-saving " (owing to the failure of income distribution to adjust itself promptly enough to the fall in investment) and thus bring about a major breakdown in the process of investment and economic growth, such as occurred during the great depressions of the 1880s or the 1930s. For when rising capital/output ratios and falling profit rates cause the rate of investment to shrink at some critical speed (or below some critical level) the fall in income generated in the investment-goods industries will react unfavourably on the level of demand in the consumption-goods industries, causing a cumulative process of contraction in incomes, investment and employment.

The mechanics of our model are thus consistent both with minor fluctuations in growth rates and with major breakdowns in the process of growth, involving heavy unemployment and temporary stagnation. If the latter occurs, net investment might even become negative, so that eventual revival will be facilitated by the gradual erosion of capital as well as by the rise in

[1] On the basis of our model, we should expect an acceleration of the rate of growth of income to be followed by an acceleration in the rate of growth of capital, so that relatively fast and relatively slow rates of growth of income and capital are correlated but the movements in the latter are more sluggish than those in the former. Hence periods of accelerating growth are likely to be periods in which the capital/output ratio is falling, and periods of decelerating growth are those in which the capital/output ratio is rising. Owing to the short-period rigidities, etc., mentioned, there may, moreover, be a rising backlog of investments during periods of accelerating growth which might cause the rate of investment to continue to accelerate for some time after the rate of growth in income had begun to slow down.

the TT' function due to the cumulative effect of unexploited new inventions. The system is liable to such major breakdowns in the wake of exploitation of major discoveries when the economy geared to a higher rate of progress needs to be " switched back " to a more moderate rate of growth; and they should be heralded by a period during which the growth in income lags behind the growth in capital, and the capital/output ratio is rising. But on the assumptions of the model presented here the occurrence of such major breakdowns is neither regular nor inevitable—the most that one can say is that the same forces that are capable of maintaining continued growth at full employment when the underlying factors making for technical change are reasonably stable are liable to break down in their operation when, as a result of instability of these factors, the situation demands a slowdown in the growth of income and capital.

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