Exercice 2 (TD 2)

(1) $\frac{\dot{A}(t)}{A(t)} = x \quad \forall x$

(2) $\dot{\hat{k}}(t) = ?$

$$\hat{k}(t) = \frac{K(t)}{A(t)L(t)}$$

$$\dot{\hat{k}}(t) = \frac{\dot{K}(t)A(t)L(t) - K(t)[\dot{A}(t)L(t) + A(t)\dot{L}(t)]}{(A(t)L(t))^2}$$

$$= \frac{\dot{K}}{AL} - \hat{k}\left[\frac{\dot{A}}{A} + \frac{\dot{L}}{L}\right]$$

$$\dot{\hat{k}} = \frac{\dot{K}}{AL} - (n+x)\hat{k}$$

$$\Leftrightarrow \quad \dot{\hat{k}} = \frac{sY - \delta K}{AL} - (n+x)\hat{k}$$

$$\Leftrightarrow \quad \dot{\hat{k}} = s\hat{y} - \delta\hat{k} - (n+x)\hat{k}$$

$$Y = K^{\alpha}(AL)^{1-\alpha}$$

$$\hat{y} = \frac{Y}{AL} \qquad \hat{y} = \frac{K^{\alpha}(AL)^{1-\alpha}}{AL}$$

$$\hat{y} = \left(\frac{K}{AL}\right)^{\alpha}\left(\frac{AL}{AL}\right)^{1-\alpha} = \hat{k}^{\alpha}$$

$$\Rightarrow \quad \dot{\hat{k}} = \underbrace{s\hat{k}^{\alpha}}_{\text{investissement par tête efficace}} - \overbrace{(n+x+\delta)\hat{k}}^{\text{dépréciation de } \hat{k}}$$

(3) A l'état stationnaire $\dot{\hat{k}} = 0$

$$\Rightarrow \quad s\hat{k}^{*\alpha} = (n+x+\delta)\hat{k}^*$$

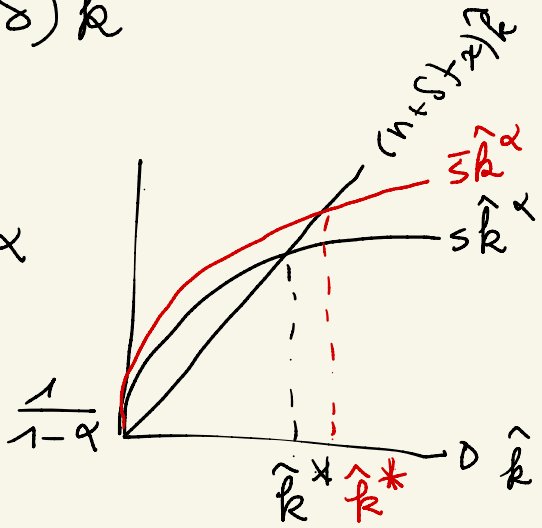
On cherche $\hat{k}^* > 0$

$$\frac{s}{n+x+\delta} = \hat{k}^{*1-\alpha}$$

$$\hat{k}^* = \left(\frac{s}{n+x+\delta}\right)^{\frac{\alpha}{1-\alpha}}$$

$$\hat{y}^* = \hat{k}^{*\alpha} = \left(\frac{s}{n+x+\delta}\right)^{\frac{\alpha}{1-\alpha}}$$

$$\hat{c}^* = (1-s)\hat{y}^* = (1-s)\left(\frac{s}{n+x+\delta}\right)^{\frac{\alpha}{1-\alpha}}$$



Variables par tête :

$$\dot{k} = \left(\frac{K}{L}\right)^{\cdot} = \frac{\dot{K}L - K\dot{L}}{L^2} = \frac{\dot{K}}{L} - nk$$

Or $\dot{k} = \frac{\dot{K}}{L} - nk = \frac{sK^{\alpha}(AL)^{1-\alpha} - \delta K}{L} - nk$

$$\Leftrightarrow \dot{k}(t) = sk^{\alpha}(t)A(t)^{1-\alpha} - nk(t)$$

$$\frac{\dot{A}}{A} = x$$

La solution stationnariser le modèle

(4) Taux de croissance

$$g_{\hat{k}} = \frac{\dot{\hat{k}}}{\hat{k}}$$

$$\hookrightarrow g_{\hat{k}} = \frac{s\hat{k}^{\alpha} - (n+x+\delta)\hat{k}}{\hat{k}} = \frac{\text{investissement net}}{\text{par unité de capital}}$$

$$\Leftrightarrow \boxed{g_{\hat{k}} = s\hat{k}^{\alpha-1} - (n+\delta+x)} \qquad \hat{k}^{*} = \left(\frac{s}{n+x+\delta}\right)^{\frac{1}{1-\alpha}}$$

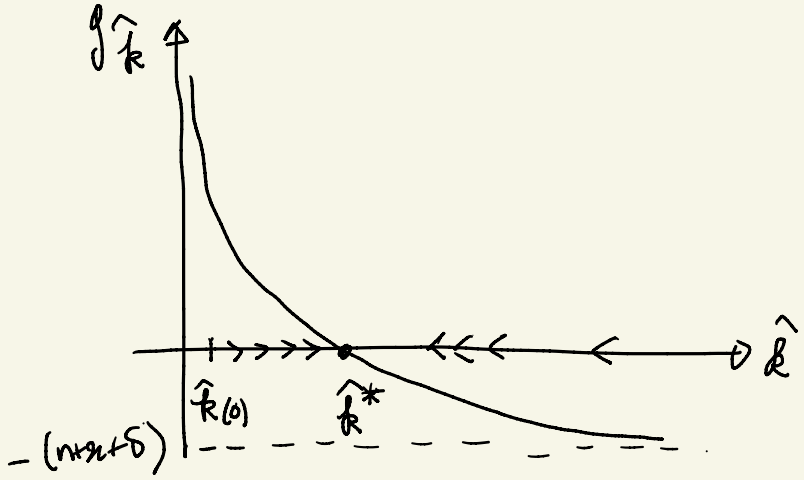
$$\hookrightarrow g_{\hat{k}} = (n+x+\delta)\left[\frac{s}{n+x+\delta} \cdot \hat{k}^{\alpha-1} - 1\right]$$

$$\hookrightarrow g_{\hat{k}} = (n+x+\delta)\left(\hat{k}^{*1-\alpha}\,\hat{k}^{\alpha-1} - 1\right)$$

$$g_{\hat{k}} = (n+x+\delta)\left(\left(\frac{\hat{k}}{\hat{k}^{*}}\right)^{\alpha-1} - 1\right)$$

si $\hat{k} < \hat{k}^* \Rightarrow \dfrac{\hat{k}}{\hat{k}^*} < 1 \Rightarrow \left(\dfrac{\hat{k}}{\hat{k}^*}\right)^{\frac{\alpha-1}{<0}} > 1$

$\Rightarrow g_{\hat{k}} > 0$



$$\frac{dg_{\hat{k}}}{d\left(\hat{k}/\hat{k}^*\right)} < 0 \quad \text{car } \alpha < 1$$

rdmt marginal du capital

(5) $\hat{y} = \hat{k}^{\alpha} \rightarrow \log \hat{y} = \alpha \log \hat{k}$

$\Rightarrow \dfrac{\dot{\hat{y}}}{\hat{y}} = \alpha \dfrac{\dot{\hat{k}}}{\hat{k}} \Leftrightarrow g_{\hat{y}} = \alpha g_{\hat{k}}$

Pendant la transition la production par tête croître $\underline{\underline{\hat{c}}}$ $\hat{k}$

$$\text{ALT} \quad g_{\hat{y}} = 0$$

On sait que $\hat{k} = \frac{K}{AL} = \frac{k}{A}$

$$\Leftrightarrow \quad k(t) = A(t)\,\hat{k}(t)$$

$$\text{Ld} \quad g_k(t) = \frac{\dot{A}(t)}{A(t)} + g_{\hat{k}} = g_{\hat{k}} + x$$

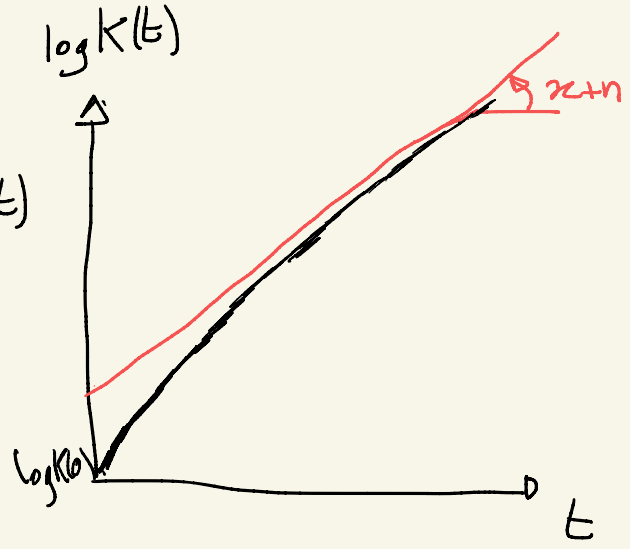
- Si l'économie est pauvre ($\hat{k} < \hat{k}^*$) alors $g_k > x$
- ——— riche ($\hat{k} > \hat{k}^*$) ——— $g_k < x$

De même $\quad y(t) = A(t)\,\hat{y}(t) \Rightarrow g_y = \alpha g_{\hat{k}} + x$

Dynamique $K(t)$?

$$K(t) = A(t) L(t) \hat{k}(t)$$

$$\Rightarrow \boxed{g_K(t) = g_{\hat{k}} + x + n}$$



$g_K > x+n$ ssi $\hat{k} < \hat{k}^*$

$$g_{\hat{y}} = \alpha g_{\hat{k}}$$

$$g_y = \alpha g_{\hat{k}} + x$$

$g_{\hat{y}}$

$g_{\hat{y}} > 0 \Rightarrow g_y > 0$

$g_{\hat{y}} < 0 \not\Rightarrow g_y < 0$

[TD3] Exercice 1

$$\dot{K} = sY - \delta K, \quad Y = K^{\alpha} L^{1-\alpha}, \quad \frac{\dot{L}}{L} = n$$

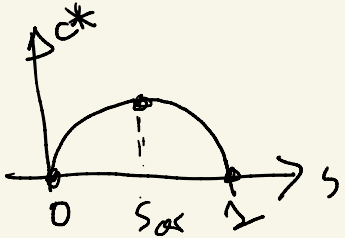
(1) $\dot{k}(t) = s k(t)^{\alpha} - (n+\delta) k(t)$

(2) $\exists ! \, k^* > 0$ tq $s k^{*\alpha} = (n+\delta) k^*$

$$\text{Ld} \quad k^* = \left(\frac{s}{n+\delta}\right)^{\frac{1}{1-\alpha}}, \quad y^* = \left(\frac{s}{n+\delta}\right)^{\frac{\alpha}{1-\alpha}}$$

$$c^* = (1-s)\left(\frac{s}{n+\delta}\right)^{\frac{\alpha}{1-\alpha}} \equiv c^*(s)$$

(3)



règle d'or $\quad s_{or} = \alpha$

$$c^*_{or} = (1-\alpha)\left(\frac{\alpha}{n+\delta}\right)^{\frac{\alpha}{1-\alpha}}$$

$$y^*_{ar} = \left(\frac{\alpha}{n+\delta}\right)^{\frac{\alpha}{1-\alpha}}$$

$$k^*_{ar} = \left(\frac{\alpha}{n+\delta}\right)^{\frac{1}{1-\alpha}}$$

$$\Rightarrow \quad k^{*1-\alpha}_{ar} = \frac{\alpha}{n+\delta}$$

$$f(k) = k^{\alpha}$$
$$f'(k) = \alpha k^{\alpha-1}$$

$$\Leftrightarrow \quad \alpha k^{*\alpha-1}_{ar} = n+\delta$$

$$\Leftrightarrow \quad \underbrace{\alpha k^{*\alpha-1}_{ar}}_{\text{rdmt marginal de } k} - \underbrace{(n+\delta)}_{\text{taux de dépréciation}} = 0$$

(4) $\tau \in [0,1]$ le taux d'imposition

$\tau Y(t)$: la ressource du gvt

$(1-\tau)Y(t)$: revenu disponible du ménage

$$\dot{K}(t) = \underbrace{s(1-\tau)Y(t)}_{\text{investissement privé}} + \underbrace{\tau Y(t)}_{\text{investissement public}} - \delta K(t)$$

$$I(t)$$

$$\dot{k} = \frac{\dot{K}}{L} - (n+\delta)k \qquad y = k^{\alpha}$$

$$\dot{k} = s(1-\tau)y + \tau y - (n+\delta)k(t)$$

$$\Leftrightarrow \dot{k} = \left[s(1-\tau)+\tau\right]k^{\alpha} - (n+\delta)k(t)$$

$$\Leftrightarrow \dot{k} = \underbrace{\left[s+\tau(1-s)\right]}_{s_\tau} k^{\alpha} - (n+\delta)k(t)$$

$$\dot{k} = s_\tau k^{\alpha} - (n+\delta)k$$

$$k^* = \left(\frac{s_\tau}{n+\delta}\right)^{\frac{1}{1-\alpha}} \qquad y^* = \left(\frac{s_\tau}{n+\delta}\right)^{\frac{\alpha}{1-\alpha}}$$

$$c(t) = (1-s)(1-\tau)\, y(t)$$

$$c^* = (1-s)(1-\tau)\left(\frac{s_\tau}{n+\delta}\right)^{\frac{\alpha}{1-\alpha}}$$

$$c^* = (1-s)(1-\tau)\left(\frac{s+\tau(1-s)}{n+\delta}\right)^{\frac{\alpha}{1-\alpha}} \equiv c^*(\tau)$$

(6)

Hypothèse : Sous accumulation

$$s < s_{or} = \alpha$$

$\bar{\tau}$ le taux d'imposition qui max la conso à LT

$$\bar{\tau} = \arg\max_{\tau} \; c^*(\tau)$$

and $\dfrac{dc^*}{d\tau} = 0$