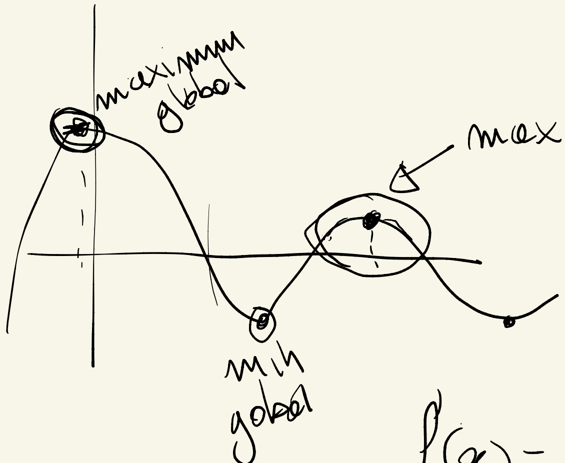
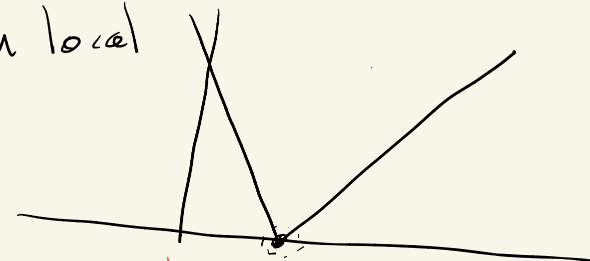
Exercice 7

$$f(x) = ax^3 + bx^2 + cx + d$$

$(a, b, c, d) \in \mathbb{R}^4$ avec $a \neq 0$





$$f'(x) = \overset{a}{\widehat{3a}}x^2 + \overset{b}{\widehat{2b}}x + \overset{c}{\widehat{c}}$$

définie $\forall x \in \mathbb{R}$

$$\Delta = b^2 - 4ac = 4b^2 - 4 \times 3 \times a \times c$$

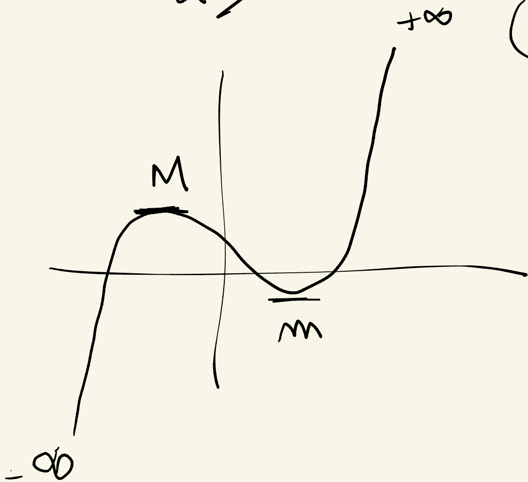
$$\Delta = 4b^2 - 12ac$$

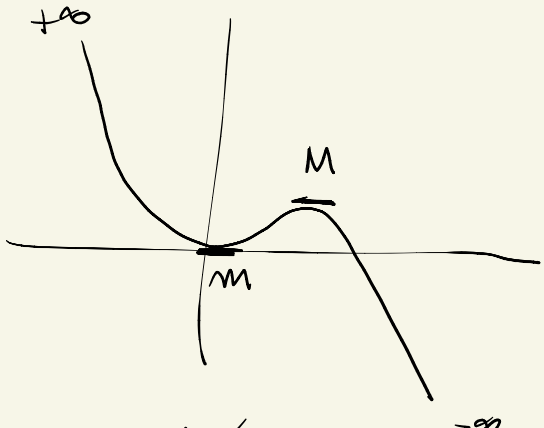
2 extrema $\iff$ 2 racines réelles distincts pour f'

$\iff \Delta > 0$

$\iff b^2 > 3ac$

$a > 0$





$a < 0$

Exercice 8 — Règle de l'Hôpital

$0/0$
∞/∞

$$\lim_{x\to\infty} \frac{f(x)}{g(x)} = \lim_{x\to\infty} \frac{f'(x)}{g'(x)} = \lim_{x\to\infty} \frac{f''(x)}{g''(x)}$$

∞ ∞ ∞ ∞

$\neq$

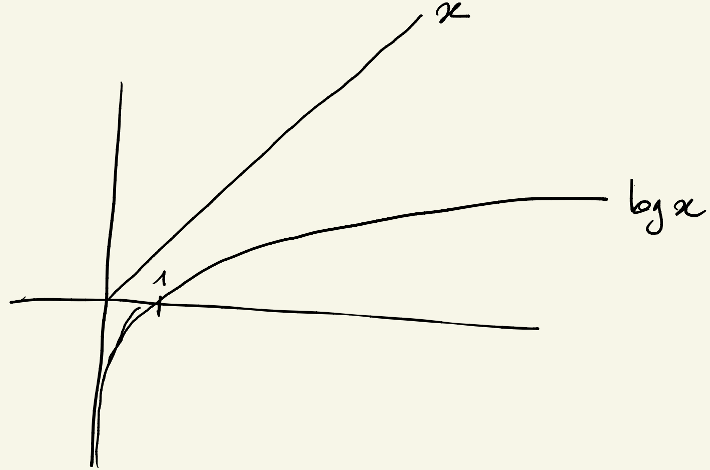
(1) $$\lim_{x\to\infty} \frac{x^2}{e^x} = \lim_{x\to\infty} \frac{(x^2)'}{(e^x)'} = \lim_{x\to\infty} \frac{2x}{e^x}$$ tjs indéterminé

$$= \lim_{x\to\infty} \frac{(2x)'}{(e^x)'} = \lim_{x\to\infty} \frac{2}{e^x} = 2 \lim_{x\to\infty} \frac{1}{e^x}$$

$$= 0.$$

(2) $$\lim_{x\to\infty} \frac{\log x}{x}$$ Forme indéterminée de type $\frac{\infty}{\infty}$

$$\lim_{x\to\infty} \frac{\log x}{x} = \lim_{x\to\infty} \frac{1/x}{1} = \lim_{x\to\infty} \frac{1}{x} = 0$$



(3) $$\lim_{x\to\infty} \frac{e^x + 3x^2}{4e^x + 2x^2} = \lim_{x\to\infty} \frac{e^x + 6x}{4e^x + 4x} = \lim_{x\to} \frac{e^x + 6}{4e^x + 4} = \lim_{x\to\infty} \frac{\cancel{e^x}}{4\cancel{e^x}} = \frac{1}{4}$$

(4) $\lim_{x\to 1} \dfrac{3x\log x}{x^2-x}$ forme indéterminée de type $0/0$

$(\overset{x}{u}\overset{\log x}{v})' = u'v + uv'$

$= 3\lim_{x\to 1} \dfrac{\log x + \frac{x}{x}}{2x-1} = 3\lim_{x\to 1} \dfrac{1+\log x}{2x-1} = 3$

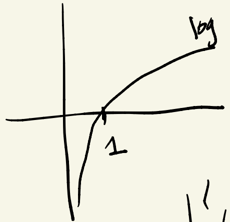
<u>Exercice 6</u> Etudions $f(x) = \dfrac{(\log x)^2_{\ u}}{x_{\ v}}$

$\left(\frac{u}{v}\right)' = \dfrac{u'v - uv'}{v^2}$

$g(f(x))' = g'(f(x))f'(x)$

Cette fonction est définie sur $\mathbb{R}_+^*$

$$f'(x) = \frac{\overset{g'}{2\log x}\cdot\overset{f'}{\frac{1}{\cancel{x}}}\cdot\cancel{x} - (\log x)^2\cdot 1}{x^2} = \frac{\log x\left[2-\log x\right]}{x^2}$$



$$\text{signe}\{f'(x)\} = \text{signe}\{\log x \cdot [2 - \log x]\}$$

	$\log x$	$2-\log x$
$x<1$	<0	>0
$x>e^2$	>0	<0

$$f'(x) = 0 \quad \text{si} \quad x = 1 \quad \text{ou} \quad x = e^2$$

$$\frac{(\log e^2)^2}{e^2} = \frac{4}{e^2}$$

x	0		1		e^2		$+\infty$
$f'(x)$		$-$	0	$+$	0	$-$	
$f(x)$	$+\infty$	$\searrow$	$\underline{\underline{0}}$ m	$\nearrow$	$\underline{4/e^2}$ M	$\searrow$	0

$$\lim_{x \to 0} \frac{(\log x)^2}{x} = \left(\lim_{x \to 0} (\log x)^2 \right) \left(\lim_{x \to 0} \frac{1}{x} \right) = +\infty$$

$$\lim_{x \to \infty} \frac{(\log x)^2}{x} = \lim_{x \to \infty} \frac{2(\log x)\frac{1}{x}}{1} = 2 \lim_{x \to \infty} \frac{\log x}{x}$$

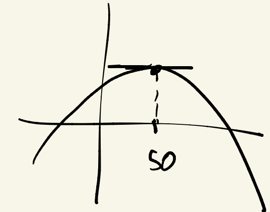
$$= 2 \lim_{x \to \infty} \frac{1/x}{1} = 2 \lim_{x \to \infty} \frac{1}{x} = 0$$

<u>Exercice 9</u> $\quad x > 0 \quad y > 0 \quad x + y = 100$

① Maximiser le produit xy

$$\Leftrightarrow \max_{x} \underbrace{x(100 - x)}_{f(x)}$$

$$f(x) = -x^2 + 100x$$



$$\hookrightarrow f'(x) = -2x + 100$$

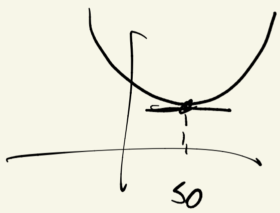
$$f'(x^*) = 0 \iff -2x^* + 100 = 0$$

$$\Rightarrow \boxed{\begin{array}{l} x^* = 50 \\ y^* = 50 \end{array}} \quad x^* = 50 > 0$$

② Minimiser $x^2 + y^2$

$$\iff \min_{\{x\}} \underbrace{x^2 + (100-x)^2}_{f(x)}$$

$$f(x) = x^2 + 100^2 + x^2 - 200x$$
$$= 2x^2 - 200x + 100^2$$

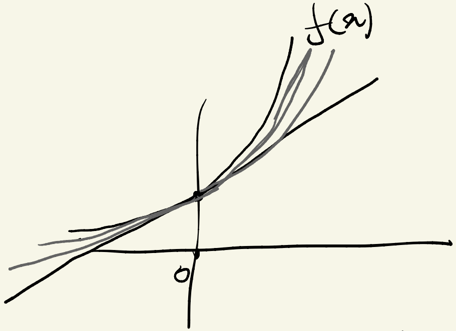


$$f'(x) = 4x - 200$$

$$f'(x^*) = 0 \iff 4x^* = 200 \iff \boxed{\begin{aligned} x^* &= 50 \\ y^* &= 50 \end{aligned}}$$

$$f^{(2)}(x) = \left(f'(x)\right)'$$

Exercice 3 & 4



$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!} (x-a)^n$$

$$(e^x)' = e^x \qquad (e^x)'' = e^x$$

$$(e^x)^{(n)} = e^x$$

En évaluant les dérivées en $x = 0$

$$f(x) = \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

définie $\forall x$
$\Rightarrow$ radius de convergence est ∞

$\left(\frac{1}{v}\right)' = -\frac{v'}{v^2}$

$$f(x) = \frac{1}{1-x} = (1-x)^{-1} \qquad f(0) = 1$$

$$f'(x) = -\frac{1}{(1-x)^2} = -(1-x)^{-2} \times (-1) = (1-x)^{-2} \qquad f'(0) = +1$$

$$f''(x) = -2(1-x)^{-3} \cdot (-1) = +2(1-x)^{-3} \qquad f''(0) = +2$$

$$f'''(x) = +3 \times 2 (1-x)^{-4} \cdot (-1) = 3 \times 2 (1-x)^{-4} \qquad f'''(0) = 3 \times 2$$

$$f''''(x) = -4 \times 3 \times 2 \times (1-x)^{-5} (-1) = 4 \times 3 \times 2 \times (1-x)^{-5} \qquad f''''(0) = 4 \times 3 \times 2$$

$$f^{(n)}(x) = n! \cdot (1-x)^{-(n+1)} \qquad f^{(n)}(0) = n!$$

Serie de Taylor $\underset{\frac{1}{1-x}}{f(x)} = \sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!} (x-0)^n = \sum_{n=0}^{\infty} x^n$ definie ssi $|x| < 1$

radius de convergence 1

$$\sum_{n=0}^{N} x^n = \frac{1-x^{N+1}}{1-x} \xrightarrow[N\to\infty]{} \frac{1}{1-x}$$

$x^{N+1} \to 0$ si $|x|<0$