Fiche de TD n°4

Exercice 1

(i) $f(x) = \log(x^2 + x^4 + 1)$ $\quad \forall x \in \mathbb{R}$

$(uv)' = u'v + uv'$

$(u+v)' = u' + v'$

$\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}$

composition : $h(x)' = f(g(x))' = f'(g(x)) \cdot g'(x)$ dérivation en chaîne

$\frac{dh}{dx} = \frac{df}{dg} \cdot \frac{dg}{dx}$

$$(e^x)' = e^x$$

$$(\log x)' = \frac{1}{x}$$

$$(\log g)' = \frac{1}{g}$$

$$\boxed{(\log u)' = \frac{u'}{u}}$$

$$f(x) = \overbrace{\log}^{f} \left(\overbrace{x^2 + x^4 + 1}^{g(x)} \right)$$

$$f'(x) = \frac{1}{x^2+x^4+1} \cdot (2x+4x^3)$$

$$f'(x) = \frac{2x+4x^3}{x^2+x^4+1}$$

$$y(x) = u(x)v(x) \qquad y'(x) = u'(x)v(x) + u(x)v'(x)$$

(ii) $$g(x) = x^2 \log(x^2 + x^4 + 1)$$

$$g'(x) = (x^2)' \log(x^2+x^4+1) + x^2 f'(x)$$

$$= 2x \cdot \log(x^2+x^4+1) + \frac{2x^3+4x^5}{x^2+x^4+1}$$

(iii) $$h(x) = e^{2x} \qquad \left(e^{u(x)}\right)' = e^{u(x)} \cdot u'(x)$$

$$(e^u)' = u'e^u$$

$$h(x) = 2e^{2x}$$

(iv) $j(x) = \log \dfrac{x^3 - 2}{x^2 + 1}$ $\leftarrow u(x)$, $\leftarrow v(x)$

$w(x)$

$$\log w(x)' = \frac{w'(x)}{w(x)}$$

$$(\log w(x))' = \frac{w'(x)}{w(x)}$$

$$w(x) = \frac{x^3 - 2}{x^2 + 1} = \frac{u(x)}{v(x)}$$

$$w'(x) = \frac{u'(x)v(x) - u(x)v'(x)}{v(x)^2}$$

$$w'(x) = \frac{3x^2(x^2+1) - (x^3-2)2x}{(x^2+1)^2}$$

$$\Leftrightarrow \quad w'(x) = \frac{\cancel{3}x^4 + 3x^2 - \cancel{2x^4} + 4x}{(x^2+1)^2}$$

$$\Leftrightarrow \quad w'(x) = \frac{x^4 + 3x^2 + 4x}{(x^2+1)^2}$$

$$j'(x) = \frac{w'(x)}{w(x)} = \frac{\dfrac{x^4+3x^2+4x}{(x^2+1)^2}}{\dfrac{x^3-2}{x^2+1}}$$

$$j'(x) = \frac{x^4+3x^2+4x}{x^3-2} \cdot \frac{\cancel{x^2+1}}{(x^2+1)^{\cancel{2}}} = \frac{x^4+3x^2+4x}{(x^3-2)(x^2+1)}$$

(v)

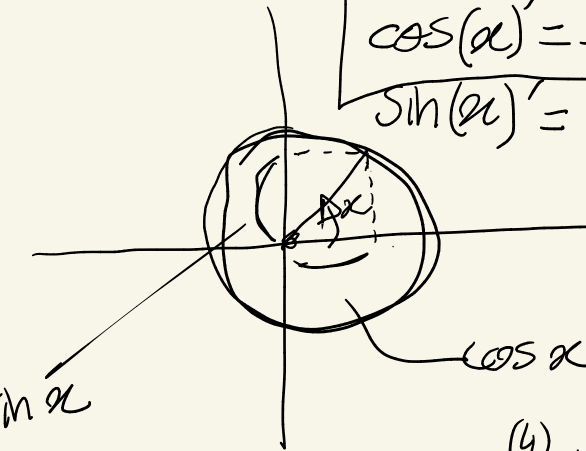
$$k(x) = \cos(\underbrace{\theta x}_{u(x)})$$

$$k'(x) = -\sin(\theta x)\cdot\theta$$

$$k'(x) = -\theta\sin(x)$$

$$\boxed{\cos(x)' = -\sin x}$$

$$\sin(x)' = \cos x$$



$$\cos^{(4)}(x) = \cos x$$

$$f(g(x))' = f'(g(x))\cdot g'(x)$$

$$\frac{df}{\cancel{dg}}\cdot\frac{\cancel{dg}}{dx}$$

Exercice 2

(1) $f(x) = e^{\overbrace{\theta x}^{u(x)}}$

$(e^u)' = u'e^u$

$f'(x) = \theta . e^{\theta x}$

la dérivée deuxième $f''(x) = \left(f'(x) \right)'$

$f''(x) = \left(\theta e^{\theta x} \right)'$

$(a f(x))' = a f'(x)$

$\lim_{x \to b} a f(x)$

$=$

$a \lim_{x \to b} f(x)$

$f''(x) = \theta \left(e^{\theta x} \right)'$

$$f''(x) = \theta\,\theta\, e^{\theta x}$$

$$f''(x) = \theta^2 e^{\theta x}$$

la dérivée troisième :

$$f'''(x) = \left(f''(x) \right)'$$

$$= \theta^2 \left(e^{\theta x} \right)'$$

$$= \theta^2 \theta\, e^{\theta x}$$

$$f'''(x) = \theta^3 e^{\theta x}$$

....

la dérivée d'ordre n

$$f^{(n)}(x) = \theta^n e^{\theta x}$$

$$\overset{?}{\Longrightarrow} f^{(n+1)}(x) = \theta^{n+1} e^{\theta x}$$

$$f^{(n+1)}(x) = \left(f^{(n)}(x)\right)' = \left(\theta^n e^{\theta x}\right)' = \theta^n \left(e^{\theta x}\right)' = \theta^n \theta e^{\theta x}$$

$$= \theta^{n+1} \cdot e^{\theta x}$$

(ii) $g(x) = \dfrac{1^{\,u}}{x_{\,v}}$

$\left(\dfrac{u}{v}\right)' = \dfrac{u'v - uv'}{v^2}$

$\dfrac{-1}{x^2}$

$\left(x^n\right)' = nx^{n-1}$

$g'(x) = -\dfrac{1^{\,u}}{x^2_{\,v}}$

$g''(x) = \left(-x^{-2}\right)' = 2x^{-3}$

$g''(x) = \dfrac{\cancel{0 \times x^2} - 1 \times 2x}{x^4} = \dfrac{2}{x^3}$

$g'''(x) = \left(2x^{-3}\right)'$

$= -3 \times 2 \times x^{-4}$

$g'''(x) = -3 \times 2 \times x^{-4}$

$g^{(4)}(x) = 4 \times 3 \times 2 \times x^{-5}$

$$g^{(5)}(x) = -5\times 4\times 3\times 2\times x^{-6}$$

$$\vdots$$

$$\boxed{g^{(n)}(x) = (-1)^{n}\, n!\, x^{-(n+1)}}$$

?

$$\Rightarrow\ g^{(n+1)}(x) = (-1)^{n+1}(n+1)!\, x^{-(n+2)}$$

$$g^{(n+1)}(x) = \left(g^{(n)}(x)\right)' = \left((-1)^{n}\, n!\, x^{-(n+1)}\right)'$$

$$g^{(n+1)}(x) = (-1)^n n! \left(x^{-(n+1)}\right)'$$

$$= (-1)^n n! \, (-1)(n+1) \, x^{-(n+1)-1}$$

$n! = n \times (n-1) \times (n-2) \times \cdots \times 2$

$$= (-1)^{n+1} (n+1)! \, x^{-(n+2)}$$

(iii) $h(x) = \log x \longrightarrow h'(x) = \frac{1}{x} = g(x)$

$h''(x) = g'(x)$

$$h'''(x) = g''(x)$$

$$\vdots$$

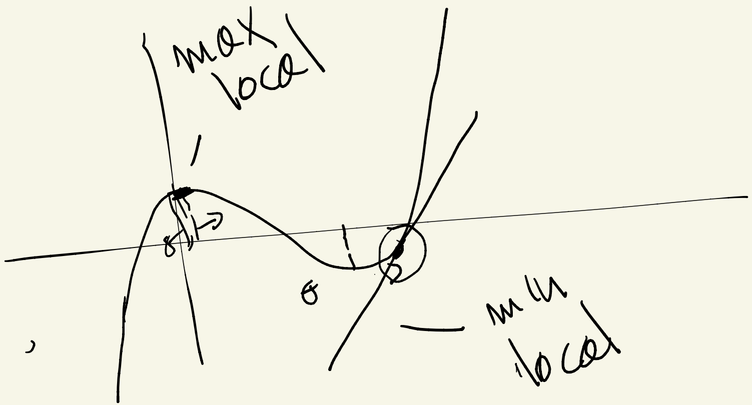
$$h^{(n)}(x) = g^{(n-1)}(x)$$

$$\boxed{h^{(n)}(x) = (-1)^{n-1} \cdot (n-1)!\, x^{-n}}$$

Exercice 3

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

Exercice 5



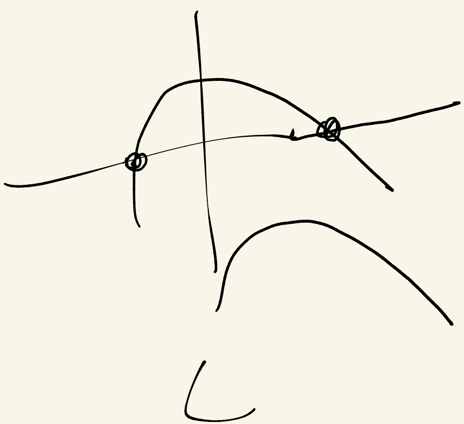
$$= x^3\left(-1 + \frac{1}{x} + \frac{2}{x^2}\right)$$

$$f(x) = -x^3 + x^2 + 2x$$

$$f'(x) = -3x^2 + 2x + 2$$

$$f \text{ croissante} \iff f' \geq 0$$

$$\iff \underbrace{-3x^2+2x+2}_{P(x)} \geq 0$$



$$\Delta = 4 - 4(-3)2 = 28 > 0$$

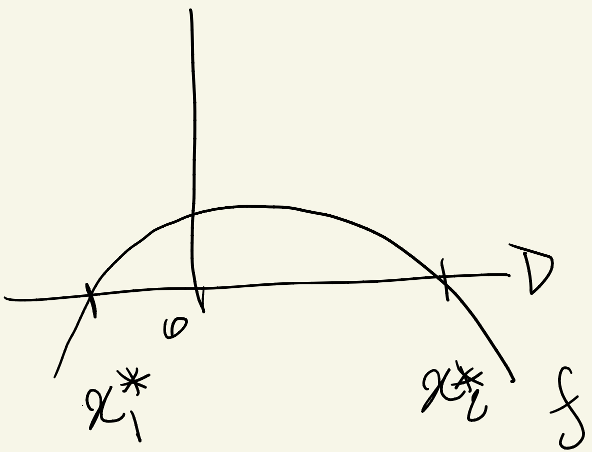
$$x^* = \frac{-2 \pm \sqrt{\Delta}}{-6}$$

$$x^* = \frac{+2 \pm \sqrt{4 \times 7}}{6}$$

$$x^* = \frac{2 \pm 2\sqrt{7}}{6}$$

$$x^* = \frac{1 \pm \sqrt{7}}{3}$$

$\sqrt{7} \simeq 2{,}64$



$$\forall x \in \Big[\underbrace{\frac{1-\sqrt{7}}{3}}_{x_1^*}, \underbrace{\frac{1+\sqrt{7}}{3}}_{x_2^*}\Big] \quad f'(x) \geq 0$$

la fonct° f est croissante sur $x \in [x_1^*, x_2^*]$

x	$-\infty$		x_1^*		x_2^*		$+\infty$
f'		$-$	0	$+$	0	$-$	
f	$+\infty$	$\searrow$	$f(x_1^*)$	$\nearrow$	$f(x_2^*)$	$\searrow$	$-\infty$

$(x_1^*, f(x_1^*))$ minimum local

$(x_2^*, f(x_2^*))$ maximum local

$$f(x_1^*) = -\left(\frac{1-\sqrt{7}}{3}\right)^3 + \left(\frac{1-\sqrt{7}}{3}\right)^2 + 2\left(\frac{1-\sqrt{7}}{3}\right) \simeq -0{,}63$$

$f(x_2^*) \simeq 2{,}11$