## Exercice 3

$$u_n = \frac{n+2}{n}$$

(1)

$$u_1 = 3, \quad u_2 = 2, \quad u_3 = \frac{5}{3}, \quad u_4 = \frac{3}{2}$$

(2) décroissance de $(u_n)$

$$u_n - u_{n-1} = \frac{n+2}{n} - \frac{n-1+2}{n-1} \qquad n > 1$$

$$= \frac{(n+2)(n-1) - (n+1)n}{n(n-1)}$$

$$= \frac{\cancel{n^2} - \cancel{n} + \cancel{2n} - 2 - \cancel{n^2} - \cancel{n}}{n(n-1)} = -\frac{2}{n(n-1)} < 0$$

$\Rightarrow$ décroissance.

(3) $|u_n - 1| = \left|\frac{n+2}{n} - 1\right|$

$$= \left|\frac{2}{n}\right| = \frac{2}{n} < \varepsilon$$

$$\iff n > \frac{2}{\varepsilon} \equiv N(\varepsilon)$$

$\forall \varepsilon > 0$, $\exists N(\varepsilon) = \frac{2}{\varepsilon}$ tq $\forall n > N(\varepsilon)$ on ait $|u_n - 1| < \varepsilon$

$\Rightarrow$ Convergence de $(u_n)_{n\in\mathbb{N}}$ vers $1$

$$\lim_{n\to\infty} u_n = 1$$

Exercice 4

$u_n = -n \quad \forall n \in \mathbb{N}$

$\lim_{n \to \infty} u_n = -\infty \quad \leftarrow$

$u_n < \alpha \in \mathbb{R} \iff -n < \alpha$

$\iff n > -\alpha \quad N$

$\forall \alpha \in \mathbb{R}, \exists N \in \mathbb{N}$ tq $\forall n > N$ on ait $u_n < \alpha$

$\Rightarrow$ divergence

Exercice 5

$u_n = \frac{(-1)^{n+1}}{n^2}$ c'est une alternée

$\Rightarrow$ non monotone

$|u_n - 0| = |u_n| < \varepsilon \iff \left|\frac{(-1)^{n+1}}{n^2}\right| < \varepsilon$

$$\iff \frac{1}{n^2} < \varepsilon$$

$$\iff n^2 > \frac{1}{\varepsilon} \qquad \cancel{\iff \begin{cases} n < -\frac{1}{\sqrt{\varepsilon}} \\ n > \frac{1}{\sqrt{\varepsilon}} \end{cases}}$$

$$\iff n > \frac{1}{\sqrt{\varepsilon}} \equiv N(\varepsilon)$$

$\forall \varepsilon > 0$, $\exists N(\varepsilon) = \frac{1}{\sqrt{\varepsilon}}$ tq $\forall n > N(\varepsilon)$ on ait

$$|u_n - 0| < \varepsilon$$

$$\iff \lim_{n \to \infty} u_n = 0$$

## Exercice 6

$$S(p): \quad q = 1 + p$$

$$D(p): \quad q = 2 - p$$

(1) Prix d'équilibre $p^* > 0$ est tel que

$$1 + p^* = 2 - p^*$$

$$\iff 2p^* = 1$$

$$\iff \boxed{p^* = \frac{1}{2}} \implies \boxed{q^* = \frac{3}{2}}$$

(2) $$p_{t+1} = p_t + \alpha\left(D(p_t) - S(p_t)\right) \qquad \alpha > 0$$

Si excés de demande $D(p_t) > S(p_t)$

alors $p_{t+1} > p_t$

On cherche le point fixe $\bar{p}$ :

$$\bar{p} = \bar{p} + \alpha\left(D(\bar{p}) - S(\bar{p})\right)$$

$$\Leftrightarrow \bar{p} = \bar{p} + \alpha\left[2 - \bar{p} - 1 - \bar{p}\right]$$

$$\Leftrightarrow \cancel{\bar{p}} = \cancel{\bar{p}} + \alpha\left[1 - 2\bar{p}\right]$$

$$\Leftrightarrow \quad \alpha\left[1-2\bar{p}\right]=0$$

$$\Leftrightarrow \quad 1-2\bar{p}=0$$

$$\Leftrightarrow \quad \bar{p}=\frac{1}{2}=p^*$$

(3) $p_0 \neq \bar{p}$

$$p_t = p_{t-1} + \alpha\left[D(p_{t-1}) - S(p_{t-1})\right]$$

$$\Leftrightarrow p_t = p_{t-1} + \alpha\left[2 - p_{t-1} - 1 - p_{t-1}\right]$$

$$\Leftrightarrow p_t = p_{t-1} + \alpha - 2\alpha p_{t-1}$$

$$\Leftrightarrow \boxed{p_t = \alpha + (1-2\alpha)p_{t-1}} \qquad \forall t$$

$$p_{t-1} = \alpha + (1-2\alpha)p_{t-2}$$

$$\Leftrightarrow \quad p_t = \alpha + (1-2\alpha)\left[\alpha + (1-2\alpha)p_{t-2}\right]$$

$$\Leftrightarrow \quad p_t = \alpha\left[1 + (1-2\alpha)\right] + (1-2\alpha)^2 p_{t-2}$$

$(1-2\alpha)^0$

$p_{t-2} = \alpha + (1-2\alpha)\,p_{t-3}$

$$\Leftrightarrow \quad p_t = \alpha\left[1 + (1-2\alpha) + (1-2\alpha)^2\right] + (1-2\alpha)^3 p_{t-3}$$

$$\hookrightarrow \quad \boxed{p_t = \alpha \sum_{i=0}^{t-1} (1-2\alpha)^i + (1-2\alpha)^t p_0}$$

$p_{t-t}$

$$\alpha + (1-2\alpha)p_{t-1} = \alpha + (1-2\alpha)\left[\alpha \sum_{i=0}^{t-2}(1-2\alpha)^i + (1-2\alpha)^{t-1}p_0\right]$$

(4) $0 < \alpha < 1/2 \iff 0 < \underbrace{1-2\alpha}_{r} < 1$

$$p_t = \alpha \underbrace{\sum_{i=0}^{t-1} r^i}_{\xrightarrow{t\to\infty}\ \frac{1}{1-r}} + \underbrace{r^t}_{\xrightarrow{t\to\infty}\ 0} p_0$$

$$r^0 + r^1 + r^2 + r^3 + \dots + r^t = \frac{1-r^{t+1}}{1-r}$$

$$\frac{1}{1-r} = \frac{1}{1-(1-2\alpha)} = \frac{1}{2\alpha}$$

$$p_t \xrightarrow[t\to\infty]{} \frac{1}{2} = \bar{p} = p^*$$

$$\alpha > \frac{1}{2} \Rightarrow \underbrace{1-2\alpha} < -1$$

## Exercice 7

1. $$\lim_{x \to \infty} \frac{2x+5}{x^2-3} = 0$$

2. $$\lim_{x \to \infty} \frac{x^3-4x^2+8}{x^2+6} = \infty$$

$$\hookrightarrow \lim_{x \to \infty} \frac{(x^3-4x^2+8)/x^2}{(x^2+6)/x^2} = \lim_{x \to \infty} \frac{x-4+8/x^2}{1+6/x^2} = \infty$$

3. $$\lim_{x \to \infty} \frac{ax^2+bx+c}{kx^2+lx+m} = \lim_{x \to \infty} \frac{a+b/x+c/x^2}{k+l/x+m/x^2} = \frac{\lim_{x \to \infty} a+b/x+c/x^2}{\lim_{x \to \infty} k+l/x+m/x^2}$$

$$= \frac{a}{k}$$

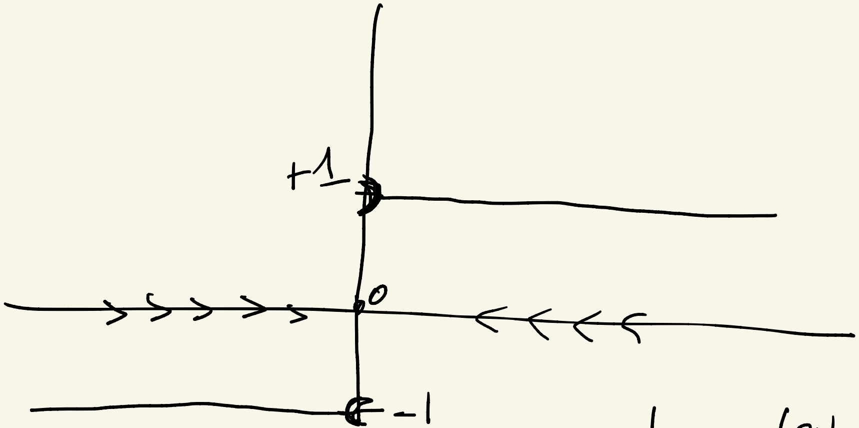
4 – $\lim\limits_{x \to -4} \dfrac{x^2-16}{x+4}$

$f(x)$ fonction rationelle sur $\mathbb{R} \setminus \{-4\}$

$\to \lim\limits_{x \to -4} \dfrac{(x-4)\cancel{(x+4)}}{\cancel{x+4}} = \lim\limits_{x \to -4} (x-4) = -8$

5 – $f(x) = \dfrac{|x|}{x}$ fonction signe

$$f(x) = \begin{cases} \dfrac{-x}{x} = -1 & \text{si } x < 0 \\ \dfrac{x}{x} = 1 & \text{si } x > 0 \end{cases}$$




$$\lim_{x\to 0^+} \frac{|x|}{x} = \lim_{x\to 0^+} \frac{x}{x} = \lim_{x\to 0^+} 1 = 1$$

$$\lim_{x\to 0^-} \frac{|x|}{x} = \lim_{x\to 0^-} \frac{-x}{x} = \lim_{x\to 0^-} -1 = -1$$

Les limites à droite et à gauche sont differentes

$\Rightarrow$ $f(x)$ n'a pas de limite en 0

$\Rightarrow$ $f(x)$ n'est pas continue en 0

## Exercice 8

1. $f(x) = 4x^2+3 \longrightarrow f'(x)$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \quad \forall x$$

$$f'(x) = \lim_{h \to 0} \frac{4(x+h)^2+3-4x^2-3}{h}$$

$$= \lim_{h \to 0} \frac{4(\cancel{x^2}+2hx+h^2)\cancel{+3}\cancel{-4x^2}\cancel{-3}}{h}$$

$$= \lim_{h \to 0} \frac{8hx+4h^2}{h} = \lim_{h \to 0} 8x + \lim_{h \to 0} 4h$$

$$= 8x + 4\left(\lim_{h \to 0} h\right) = 8x$$

2- $f(x) = x^n$ $\qquad f'(x) = ?$

$$f'(x) = \lim_{h \to 0} \frac{(x+h)^n - x^n}{h}$$

Binôme de Newton

$$(a+b)^2 = a^2 + 2ab + b^2$$
$$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$

$$(a+b)^n = \sum_{i=0}^{n} C_n^i a^{n-i} b^i$$

$n! = n \times (n-1) \times (n-2) \times \cdots \times 2 \times 1$

avec $\underbrace{C_n^i = \frac{n!}{i!(n-i)!}}$ // coefficient binomiale

$$f'(x) = \lim_{h \to 0} \frac{\sum_{i=0}^{n} C_n^i x^{n-i} h^i - x^n}{h}$$

Convention

$$C_n^0 = \frac{n!}{0!\, n!} = 1 \qquad 0! = 1$$

$$f'(x) = \lim_{h \to 0} \frac{\cancel{x^n h^0} + \sum_{i=1}^{n} C_n^i x^{n-i} h^i \cancel{- x^n}}{h}$$

$$= \lim_{h \to 0} \sum_{i=1}^{n} C_n^i x^{n-i} h^{i-1}$$

$$= \lim_{h \to 0} \left( C_n^1 x^{n-1} h^0 + \sum_{i=2}^{n} C_n^i x^{n-i} h^{i-1} \right)$$

$$= C_n^1 x^{n-1} + 0$$

$$= \frac{n!}{1! \cdot (n-1)!} \cdot x^{n-1} = \frac{n \times \cancel{(n-1)} \times \cancel{(n-2)} \times \cdots \times \cancel{2} \times \cancel{1}}{\cancel{(n-1)} \times \cancel{(n-2)} \times \cdots \times \cancel{2} \times \cancel{1}} \cdot x^{n-1}$$

$$\boxed{f'(x) = n x^{n-1}}$$